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A
TREATISE
OF
PRACTICAL SURVEYING;

Which is demonſtrated
From its Firſt PRINCIPLES.

WHEREIN

Every Thing that is Uſeful and Curious in that
ART is fully conſidered and explained.

PARTICULARLY

Four new and very conciſe Methods to determine the Areas of Right-lined Figures Arithmetically, or by Calculation, as well as the Geometrical ones heretofore treated of; with two other new Geometrical Methods much more accurate and ready than any of the former, never before made public.

ALSO

The Method of Tracing Defaced Mearings from
the Down (or any other) Survey.

Very uſeful to Perſons who have any Property in Land, to Lawyers in controverted Surveys, and to Practical Surveyors.

The whole illuſtrated with Copper-Plates.

The SECOND EDITION.

By ROBERT GIBSON, Teacher of the Mathematics.

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MVSEVM
BRITAN
NICVM

TO

THE RIGHT HONOURABLE

THOMAS CARTER, Eſq;

MASTER of the ROLLS, &c. &c.

SIR,

THE great Knowledge You have acquired in all kinds of Polite Literature, and the Encouragement You conſtantly give for the Propagation of them in general, but more particularly to the Mathematical Sciences, are Motives ſufficient to direct me to dedicate this Work to you.

Panegyrick, SIR, is not my Province, and if it were, I know I muſt offend one of your extenſive Learning, ſhould I attempt to diſplay it on this Occaſion: Yet I muſt ſay, that if but one fourth Part of the Nobility and Gentlemen of opulent Fortunes in this Kingdom, were of your unlimited generous Diſpoſition in encouraging Arts and Sciences; we might in a few Years, be able to vie with any Country whatſoever.

That you may long enjoy an uninterrupted State of good Health is the Wiſh of moſt; but of none more particularly, than of SIR,

Your moſt obliged
moſt obedient,
and moſt aſſured
Humble Servant,

ROBERT GIBSON.

PREFACE.

THE Word *Geometry* imports no more than *to meaſure the Earth*, or to meaſure Land; yet in a larger and more proper Senſe, it is applied to all Sorts of Dimenſions. It is generally ſuppoſed to have had its Riſe among the *Egyptians*, from the River *Nile's* deſtroying and confounding all their Land-marks by its annual Inundations, which laid them under the Neceſſity of inventing certain Methods and Meaſures, to enable them to diſtinguiſh and adjuſt the Limits of their reſpective Grounds, when the Waters were withdrawn. And this Opinion is not entirely to be rejected, when we conſider that *Moſes* is ſaid to have acquired this Art, when he reſided at the *Egyptian* Court. And *Achilles Tatius* in the Beginning of his Introduction to *Aratus*'s *Phænomena*, informs us, that the *Egyptians* were the firſt who meaſured the Heavens and the Earth (and of courſe the Earth firſt) and that their Science in this Matter, was engraven on Columns, and by that means delivered to Poſterity.

It is a Matter of ſome Wonder, that though Surveying appears to have been the firſt, or at leaſt one of the firſt, of the Mathematical Sciences, that the reſt have met with much greater Improvements from the Pens of the moſt eminent Mathematicians,

maticians, while this ſeems to have been neglected; inſomuch that I have not been able to meet with one Author, who has ſufficiently explained the whole Art in its Theory and Practice: For the moſt part it has been treated of in a practical Manner only; and the few who have undertaken the Theory, have in a great Meaſure omitted the Practice.

Theſe Conſiderations induced me to attempt a methodical, eaſy and clear Courſe of *Surveying*; how far I have ſucceeded in it, muſt be determined by the impartial Reader: The Steps I have taken to render the whole evident and familiar are as follow:

In Section the firſt, you have Decimal Fractions, the Square Root, Geometrical Definitions, ſome neceſſary Theorems and Problems; with the Nature and Uſe of the Tables of Logarithm Numbers, Sines, Tangents, and Secants.

The ſecond Section contains Plane Trigonometry right angled and oblique, with its Application in determining the Meaſures of inacceſſible Heights and Diſtances.

The third Section gives an Account of the Chains and Meaſures uſed in *Great-Britain* and *Ireland*. Methods of Surveying and of taking inacceſſible Diſtances by the Chain only, with ſome neceſſary Problems; alſo a particular Deſcription of the ſeveral Inſtruments uſed in Surveying, with their reſpective Uſes.

The fourth Section contains five various Methods of finding the Areas of Maps, from their Geometrical Conſtruction; two of which more conciſe than the reſt, were never before made public.

The fifth Section contains four new, and much more conciſe Methods of determining the Areas of Surveys from the Field Notes, or by Calculation than any hitherto publiſhed; and I venture to aſſert that it is impoſſible (from the Nature of right-lined Figures) that any Method or Methods more conciſe than theſe can be inveſtigated.

To theſe Methods is annexed a ſhort Table of Difference of Latitude and Half Departure, to every Degree and Quarter of a Degree of the Quadrant, the ſtationary Diſtance being one Chain; which will be found as ready, by a little Practice, and perhaps more exact, than thoſe already publiſhed. To this is annexed a Table for reducing Degrees of the *Circumferentor* to thoſe of the *Quarter Compaſs*, and the contrary; alſo the Method of changing Angles of the Field, taken by *Theodolite*, *Semicircle*, or *Plane Table*, to thoſe from the Meridian; for the greater Readineſs and Accuracy in Protraction, as well as to prepare them for Calculation.

Truth calls upon me to acknowledge, that the Methods by Calculation, herein ſet forth, got their Riſe from thoſe of the late *Thomas Burgh*, Eſq; who firſt diſcovered an univerſal Method for determining the Areas of right-lined Figures, and for which he obtained a parliamentary Reward. I hope therefore it cannot be conſtrued as an Intention in me to take from his great Merit, when I ſay, that the Methods herein contained, are much more conciſe and ready than his.

Section the ſixth contains the Nature of Off-ſets, and the Method of caſting them up by the Pen: The Nature and Application of Interſections: The Methods of enlarging, diminiſhing, and connecting of Maps: The Method of tracing defaced Mearings from the Down (or any other) Surveys: The Variation of the Compaſs by Amplitudes and Azimuths, with ſome of its Uſes; to which is added, a Table of the Sun's Declination: The Method of reducing one Meaſure to another; how to find by what Scale a Map is laid down, having the Map and Area given: How to find the Content of Ground that is ſurveyed by a Chain that is too long or too ſhort: The Method of dividing Lands: And the whole concludes with ſome neceſſary Directions and Remarks on Surveys in general.

THE PRINCIPLES OF SURVEYING,

SECT. I.

Containing Decimal Fractions, the Square Root, Geometrical Definitions, Theorems and Problems; with the Nature and Uſe of the Tables of Logarithm Numbers, Sines, Tangents, and Secants.

DEFINITION.

SURVEYING is that Art which enables us to give a Plan, or juſt Repreſentation, of any Piece or Parcel of Land, and to determine the Content thereof, in ſuch Meaſure as is agreeable and cuſtomary to the Country or Place where the Land is.

This Science depends on ſome Part of the Mathematics, which muſt be known before we can treat of it, wherefore we ſhall begin with

DECIMAL FRACTIONS.

If we ſuppoſe Unity or any one Thing to be divided into any aſſigned Number of equal Parts, this Number is called the Denominator; and if we chuſe to take any Number of ſuch Parts leſs than the Whole, this is called the Numerator of a Fraction.

The Numerator, in the Vulgar Form, is always wrote over the Denominator, and theſe are ſeparated by a ſmall Line thus $\frac{5}{12}$ or $\frac{7}{12}$ $\frac{\text{Numerator}}{\text{Denominator}}$ the firſt of theſe is called 5 Twelfths, and the latter 7 Twelfths of an Inch, Yard, Perch, *&c.* or of whatever the whole Thing originally was.

Fractions are expreſſed in two Forms, that is either vulgarly or decimally.

All Fractions whoſe Denominators do not conſiſt of a Cypher or Cyphers ſet after Unity, are called Vulgar ones, and their Denominators are always wrote under their Numerators. The treating of theſe would be foreign to our preſent Purpoſe. But Fractions whoſe Denominators conſiſt of an Unit, prefixed to one or more Cyphers, are called Decimal Fractions; the Numerators of which are wrote without their Denominators, and are diſtinguiſhed from Integers by a Point prefixed: Thus $\frac{2}{10}$ $\frac{42}{100}$ and $\frac{172}{1000}$ in the Decimal Form, are expreſſed by .2 .42 .172.

The Denominators of ſuch Fractions always conſiſting of an Unit, prefixed to as many Cyphers as there are Places of Figures in the Numerators, it follows, that any Number of Cyphers put after thoſe Numerators, will neither increaſe or leſſen their Value: For $\frac{3}{10}$ $\frac{30}{100}$ and $\frac{300}{1000}$ are all of the ſame Value, and will ſtand in the Decimal Form thus .3 30 .300; but a Cypher or Cyphers prefixed to thoſe Numerators, leſſen their Value in a tenfold Proportion: For $\frac{3}{10}$ $\frac{3}{100}$ and $\frac{003}{1000}$ which in the Decimal Form we denote by .3 .03 and .003, are Fractions, of which the firſt is ten Times greater than the ſecond; and the ſecond, ten Times greater than the third.

Hence it appears, that as the Value and Denomination of any Figure or Number of Figures in common Arithmetic is enlarged, and becomes ten or a hundred, or a thouſand Times greater, by placing one or two, or three Cyphers after it; ſo in decimal Arithmetic, the Value of any Figure or Number of Figures, decreaſes, and becomes ten, or a hundred, or a thouſand Times leſs, while the Denomination of it increaſes, and becomes ſo many Times greater, by prefixing one, or two, or three Cyphers to it: And that any Number of Cyphers, before an Integer, or after a decimal Fraction, have no Effect in changing their Values.

Integers.							Decimals.					
1	2	7	1	6	3	.	5	1	2	3	6	7
Hund. of Thouſ.	Tens of Thouſ.	Thouſands	Hundreds	Tens	Units		Tenths	Pts of an Hund.	Pts of a Thouſ.	Pts of Ten Thouſ.	Pts of Hun. Th.	Pts of a Million

Addition of DECIMALS.

Having placed thoſe Figures which are equidiſtant from the Point, (as well Integers as Fractions) under each other, add them as if they were Integers.

EXAMPLES.

Add 4.7832 3.2543 7.8251 6.03 2.857 and 3.251 together. Place them thus,

4.7832
3.2543
7.8251
6.03
2.857
3.251

Anſwer 28.0006

Add 6.2 121.306 .75 2.7 and .0007 together.

121.306
.75
2.7
.0007

Anſwer 130.9567

What’s the Sum of 6.57 1.026 .75 146.5 8.7 526 3.97, and .0271?

Anſwer 693.5431.

What’s the Sum of 4.51 146.071 .507 .0006 132 62.71 .507 7.9 and .10712?

Anſwer 354.31272.

Subſtraction of DECIMALS.

Having placed the Figures which are equidiſtant from the Point, under each other; deduct as if they were Integers.

EXAMPLES.

From 38.765 take 25.3741
25.3741

Anſwer 13.3909

From 2.4 take .8472
.8472

1.5528

From 71.45 take 8.4837248
Anſwer 62.9662752.

From 84 take 82. 3412
Anſwer 1.6588

Multiplication of DECIMALS.

Place the Multiplicand, and Multiplier, after any Manner under each other; and having multiplied as in whole Numbers, cut off as many Places of Decimals in the Product, counting from the right Hand towards the left, as there are in the Multiplicand, and Multiplier: But if there be not a ſufficient Number of Places in the Product, the Defect may be ſupplied, by prefixing Cyphers thereto.

For the Denominator of the Product, being an Unit, prefixed to as many Cyphers, as the Denominators of the Multiplier and Multiplicand contain of Cyphers, it follows, that the Places of Decimals in the Product, will be as many as in the Numbers from whence it aroſe.

EXAMPLES.

```
Multiply 48.765 by .003609
            .003609
           --------
             438885
            292590
           146295
           ---------
Anſwer .175992885
       ----------

Multiply .121
      by .14
         ----
          484
         121
         ----
Anſwer .01694
```

Multiply 121.6 by 2.76
2.76

7296
8512
2432

Anſwer 335.616

Multiply .0089789 by 1085
Anſwer 9.7421065

Multiply .248723 by .13587
Anſwer .0337939940l

Diviſion of DECIMALS.

Having divided as in whole Numbers, annexing Cyphers to the Dividend if they be wanted; the Decimal Places in the Diviſor and Quotient muſt be equal to thoſe in the Dividend, and the Defect ſupply'd by prefixing Cyphers to the Quotient.

For the Dividend is a Product, contained under the Diviſor and Quotient; and that Product contains as many Places of Decimals as the Numbers do from whence it aroſe: Therefore the Difference between the Number of Decimals in the Dividend, and Diviſor, muſt be cut off in the Quotient.

EXAMPLES.

Divide .144 by .12
.12).144(1.2

24

EXAMPLES.

Divide 63.72413456922 by 2718.

2718)63.72413456922(.02344522979

9364

12101

12293

14214

6245

8096

26609

21472

24462

Anſwer .02344522979.

There being 11 decimal Figures in the Dividend, and none in the Diviſor, 11 Figures are to be cut off in the Quotient; but as the Quotient itſelf conſiſts of but 10 Figures, we prefix to them a Cypher to compleat that Number.

Divide 1.728 by .012

.012)1.728(144

52

48

Becauſe the Number of decimal Figures in the Diviſor and Dividend, are alike, the Quotient will be Integers.

Divide 2.00000 by 3.1416
3.1416)2.00000(.636618
115040
207920
194240
57440
260240
8912

There being 4 decimal Figures in the Diviſor, and 10 including the Cyphers brought down in the Dividend, the Difference, which is 6 Figures, to be cut off in the Quotient.

Divide 87446071 by .004387.
Anſwer 199.33.
Divide .624672 by 482.
Anſwer .001296.
Divide 66.993548 by 27.4.
Anſwer 2.44502.

PROB. I.

To Reduce a Vulgar Fraction to a decimal one of the ſame Value.

Having annex'd a ſufficient Number of Cyphers as Decimals, to the Numerator of the Vulgar Fraction, divide by the Denominator; and the Quotient thence ariſing, will be the decimal Fraction required.

EXAM-

EXAMPLES.

Reduce $\frac{3}{4}$ to a Decimal Fraction.

4)3.00(.75 Anſwer.

20

For $\frac{3}{4}$ of one Shilling, Yard, Perch, *&c.* is equal to one Fourth of three Shillings, Yards, Perches, *&c.* therefore if three be divided by 4, the Quotient will be the Anſwer.

Reduce $\frac{2}{5}$ to a Decimal Fraction.

5)2.0(.4 Anſwer.

Reduce $\frac{12}{25}$ to a Decimal Fraction.

25)12.00(.48 Anſwer.

200

Reduce $\frac{25}{218}$ to a Decimal Fraction.

Anſwer .1146789.

PROB. II.

To find the Value of a Decimal Fraction, in the known Parts of the Integer.

Multiply the Decimal propoſed into the Number of equal Parts contained in the Integer, and the Product will be the Number of ſuch Parts as are expreſſed by the Fraction.

What's the Value of .25 of a Pound Sterling?

20

Anſwer Shillings 5.00

For .25 or $\frac{25}{100}$ of one Pound, is equal to the one-hundredth Part of 25 Pounds, or of the Shillings in 25 Pounds, which are 500; therefore the one-hundredth Part thereof will be 5 Shillings; which is effected by cutting off the two Cyphers, for the two Decimals, by a Point.

What's the Value of .385 of a Pound Sterling?
20
Shillings 7.700
12
Pence 8.400
4
Farthings 1.600

What's the Value of .48 of a Chain of 50 Links?
50
Anſwer, Links 24.00

What's the Value of .2864 of a Shilling?
12
Pence 3.4368
4
Farthings 1.7472

What's the Value of .287 of a Pound Weight, Troy?

	Oz.	dwt.	grs.
Anſwer	3.	8.	21.

What's the Value of .2945 of a Pound Avoirdupoiſe?

	Oz.	dwt.
Anſwer	4	11 ¼

The EXTRACTION of the SQUARE ROOT.

A Square Number is the Product of a Number multiplied by itſelf; and the Number ſo multiplied is called the Root of that Square, thus 9 is the Square of 3, and 3 is the Root of 9, for 3 multiplied by 3 is 9.

If a Square Number be given to find it's Root, obſerve if the Number of Figures or Places in the given Square be odd or even, if they be odd, find the Root of the firſt Figure, but if they be even, of the two firſt; under which place the Square of that Root, and deduct, placing the Root in the Quotient, and bring down two Figures to the Remainder.

Let the double of the ſaid Root be made a Diviſor to all the Figures of that laſt Remainder, except the laſt; put the Quotient thereof with the Root, or former Quotient; and having multiplied it into the Numbers ſo formed, deduct the Product from the foregoing Figures or Reſolvend: And in like Manner proceed, 'till all the Figures of the given Square are exhauſted.

If there be any Decimals in the given Square, their Number muſt be even or made ſo, before we begin to find the Root, by adding a Cypher to the right Hand; and for every two Places of Decimals in the Square, let one be cut off in the Root.

EXAMPLES.

1. What's the Square Root of this Square Number 298116?

```
         29,81,16(546
         25
         ——
   104)481
        416
        ———
  1086).6516
         6516
         ————
         ....
```

Becauſe the Number of Figures in the given Square Number is even, we find the neareſt Square Number to the two firſt Figures 29, which is 25, the Root whereof 5, we ſet in the Quotient, and deduct 25 from 29, and to the Reſidue 4, we annex the following Figures 81, ſo we have 481 for a Reſolvend.

The double of the firſt Figure in the Quotient being 10 is then ſet as a Diviſor to 48, all the Figures in the Reſolvend but the laſt; and finding it to be contained 4 Times, we annex the 4 to the Diviſor and Quotient; the then Diviſor 104, is multiplied by the laſt Figure in the Quotient 4, and the Product 416 is deducted from the Reſolvend 481, to the Reſidue whereof is annexed the two following Figures in the Square, ſo have we 6516 for a new Reſolvend, to all which Figures but the laſt we make 108, the double of 54, the Figures in the Quotient a Diviſor, and finding it will be contained 6 Times, we place 6 in the Diviſor and Quotient; the then Diviſor 1086 is multiplied by the laſt Figure in the Quotient 6, and the Product being ſet under the Reſolvend and thence deducted leaves Nothing: So is 546 the Root ſought.

For if the Root 546 be ſquared or multiplied by 546, the Product will be the ſquare Number given.

2. What is the Square Root of 1710864?

```
                 Root
      1,71,08,64(1308 Anſwer.
      1
     ____
   23).71
       69
     ______
 2608).20864
       20864
     ______
       .....
```

What is the Square Root of 3857.3?

Here being an odd decimal Figure, we annex any odd Number of Cyphers to make the Decimal Places even; and then extracting the Root as before, we thence cut off half the Number of Decimals that we have in the Square. Thus

```
                      Root
      3857.300000(62.107 Anſwer.
      36
     ____
  122)257
      244
     _____
 1241)1330
      1241
     ______
124207) 890000
        869449
       _______
         .20551
       _______
```

If to the Square of this Root we add the remaining Figures 20551, we will have our given Square, whoſe Root was required.

What is the Square Root of 16007.3104?
Anſwer 126.52.

What is the Square Root of 348.17320836?
Anſwer 18.6594.

What is the Square Root of 12345678987654321?
Anſwer 111111111.

The Application of this will hereafter be ſhewn.

THE ELEMENTS OF Plane Geometry.

DEFINITIONS.

Plate I.

1. GEOMETRY is that Science wherein we conſider the Properties of Magnitude.

2. A Point is that which has no Parts, being of itſelf indiviſible, as A.

3. A Line has Length but no Breadth, as AB. Figures 1 and 2.

4. The Extremities of a Line are Points, as the Extremities of the Line AB are the Points A and B. Figures 1 and 2.

5. A right Line is the ſhorteſt that can be drawn between any two Points, as the Line AB. Fig. 1. but if it be not the ſhorteſt, it is then called a curve Line, as AB. Fig. 2.

Plate I.

6. A Superficies or Surface is conſidered only as having Length and Breadth, without Thickneſs, as ABCD. Fig. 3.

7. The Extremities of a Superficies are Lines.

8. The Inclination of two Lines meeting one another (provided they do not make one continued Line) or the opening between them is called an Angle. Thus Fig. 4, the Inclination of the Line AB to the Line BC meeting each other in the Point B, or the opening of the two Lines BA and BC. is called an Angle, as ABC.

Note, When an Angle is expreſſed by three Letters, the Middle one is that at the angular Point.

10. When the Lines that form the Angle are Right ones, it is then called a Right-lined Angle, as ABC. Fig. 4. If one of them be Right and the other Curved, it is called a Mix'd-Angle, as B. Fig. 5. If both of them be curved it is called a Curved-lined or a Spherical Angle, as C. Fig. 6.

11. If a right Line CD (Fig. 7.) fall upon another right Line AB, ſo as to incline to neither Side, but make the Angles ADC, CDB on each Side equal to each other, then thoſe Angles are called right Angles, and the Line CD a Perpendicular.

12. An obtuſe Angle is that which is wider or greater than a right one, as the Angle ADE Fig. 7. and an acute Angle is leſs than a right one, as EDB. Fig. 7.

Plate I.

13. Acute and obtuſe Angles in general are called oblique Angels.

14. If a right Line CB (Fig. 8.) be faſtened at the End C, and the other End B be carried quite round, then the Space comprehended is called a Circle; and the curve Line deſcribed by the Point B is called the Circumference or the Periphery of the Circle; the fixed Point C is called its Center.

15. The deſcribing Line CB. (Fig. 8.) is called the Semidiameter or Radius, or any Line from the Center to the Circumference: Whence all Radii of the ſame or of equal Circles are equal.

16. The Diameter of a Circle is a right Line drawn thro' the Center, and terminating on either Side of the Circumference; and it divides the Circle and Circumference into two equal Parts called Semicircles; and is double the Radius, as AB or DE. Fig. 8.

18. The Circumference of every Circle is ſuppoſed to be divided into 360 equal Parts called Degrees, and each Degree into 60 equal Parts called Minutes, and each Minute into 60 equal Parts called Seconds, and theſe into Thirds, Fourths, &c. theſe Parts being greater or leſs as the Radius is.

19. A Chord is a right Line drawn from one End of an Arc, or Arch (that is any Part of the Circumference of a Circle) to the other; and is the Meaſure of the Arc. Thus the right Line HG is the Meaſure of the Arc HBG. Fig. 8.

20. The Segment of a Circle is any Part thereof, which is cut off by a Chord: Thus the Space which is comprehended between the Chord HG

Plate I.
and the Arc HBG, or that which is comprehended between the ſaid Chord HG and the Arc HDAEG are called Segments. Whence 'tis plain, Fig. 8.

1. That any Chord will divide the Circle into two Segments.

2. The leſs the Chord is, the more unequal are the Segments.

3. When the Chord is greateſt it becomes a Diameter, and then the Segments are equal; and each Segment is a Semicircle.

21. A Sector of a Circle is a Part thereof leſs than a Semicircle which is contained between two Radii and an Arc: Thus the Space contained between the two Radii CH, CB, and the Arc HB is a Sector. Fig. 8.

22. The right Sine of an Arc, is a perpendicular Line let fall from one End thereof, to a Diameter drawn to the other End: Thus HL is the right Sine of the Arc HB.

The Sines on the ſame Diameter encreaſe 'till they come to the Center, and ſo become the Radius: Hence it is plain that the Radius CD is the greateſt poſſible Sine, and thence is called the whole Sine.

Since the whole Sine CD (Fig. 8.) muſt be perpendicular to the Diameter (by Def. 22.) therefore producing DC to E the two Diameters AB and DE croſs one another at right Angles, and thus the

Periphery is divided into four equal Parts, as BD, DA, AE, and EB; (by Def. 11.) and ſo BD becomes a Quadrant or the fourth Part of the Periphery: Therefore the Radius DC is always the Sine of a Quadrant, or of the fourth Part of the Circle BD.

Sines are ſaid to be of as many Degrees as the Arc contains Parts of 360: So the Radius being the Sine of a Quadrant becomes the Sine of 90 Degrees, or of the fourth Part of the Circle, which is 360 Degrees.

23. The verſed Sine of an Arc is that Part of the Diameter that lies between the right Sine and the Circumference: Thus LB is the verſed Sine of Arc HB. Fig. 8.

24. The Tangent of an Arc is a right Line touching the Periphery being perpendicular to the End of the Diameter, and is terminated by a Line drawn from the Center thro' the other End: Thus BK is the Tangent of the Arc HB. Fig. 8.

25. And the Line which terminates the Tangent that is CK, is called the Secant of the Arc HB. Fig. 8.

26. What an Arc wants of a Quadrant is called the Complement thereof: Thus DH is the Complement of the Arc HB.

27. And what an Arc wants of a Semicircle is called the Supplement thereof: Thus AH is the Supplement of the Arc HB. Fig. 8.

Plate I.

28. The Sine, Tangent, or Secant of the Complement of any Arc, is called the Co-Sine, Co-Tangent, or Co-Secant of the Arc itſelf; Thus FH is the Sine, DI the Tangent, and CI the Secant of the Arc DH; or they are the Co-Sine, Co-Tangent, or Co-Secant of the Arc HB. Fig. 8.

29. The Sine of the Supplement of an Arc, is the ſame with the Sine of the Arc itſelf, for drawing them according to Def. 22. there reſults the ſelf-ſame Line; thus HL is the Sine of the Arc HB, or of its Supplement ADH. Fig. 8.

30. The Meaſure of a right-lined Angle is the Arc of a Circle ſwept from the angular Point and contained between the two Lines that form the Angle: Thus the Angle HCB (Fig. 8.) is meaſured by the Arc HB, and is ſaid to contain ſo many Degrees as the Arc HB does; ſo if the Arc HB is 60 Degrees the Angle HCB is an Angle of 60 Degrees.

Hence Angles are greater or leſs according as the Arc deſcribed about the angular Point, and terminated by the two Legs, contain a greater or leſs Number of Degrees of the whole Circle.

31. The Sine, Tangent, and Secant of an Arc, is alſo the Sine, Tangent, and Secant of an Angle whoſe Meaſure the Arc is; thus becauſe the Arc HB is the Meaſure of the Angle HCB, and ſince HL is the Sine, BK the Tangent, and CK the Secant, BL the verſed Sine, HF the Co-Sine, DI the Co-Tangent, and CI the Co-Secant, &c. of the Arch BH; then HL, is called the Sine, BK the Tangent

Plate I.

Tangent, CK the Secant, &c. of the Angle HCB, whoſe Meaſure is the Arc HB. Fig. 8.

32. Parallel Lines are ſuch as are equidiſtant from each other, as AB, CD. Fig. 9.

33. A Figure is a Space bounded by a Line or Lines. If the Lines be right it is called a rectilineal Figure, if curved it is called a curvilineal Figure; but if they be partly right and partly curved Lines, it is called a mixt Figure.

34. The moſt ſimple rectilineal Figure is a Triangle, being compoſed of three right Lines, and is conſidered in a double Capacity; 1ſt, with reſpect to its Sides; and 2d, to its Angles.

35. In reſpect to its Sides it is either Equilateral having the three Sides equal, as A. Fig. 10.

36. Or Iſoſceles, having two equal Sides, as B. Fig. 11.

37. Or Scalene having the three Sides unequal, as C. Fig. 12.

38. In reſpect to its Angles, it is either right-angled, having one right Angle as D. Fig. 13.

39. Or obtuſe angled, having one obtuſe Angle, as E. Fig. 14.

40. Or acute angled, having all the Angles acute as F. Fig. 15.

Plate I.

41. Acute and obtuſe-angled Triangles are in general called oblique angled Triangles, in all which any Side may be called the Baſe, and the other two the Sides.

42. The perpendicular Height of a Triangle is a Line drawn from the Vertex to the Baſe perpendicularly: Thus if the Triangle ABC be propoſed, and BC be made its Baſe, then if from the Vertex A the Perpendicular AD be drawn to BC, the Line AD will be the Height of the Triangle ABC, ſtanding on BC as its Baſe. Fig. 16.

Hence all Triangles between the ſame Parallels have the ſame Height, ſince all the Perpendiculars are equal from the Nature of Parallels.

43. Any Figure of four Sides is called a quadrilateral Figure.

44. Quadrilateral Figures whoſe oppoſite Sides are parallel, are called Parallelograms: Thus ABCD is a Parellelogram. Fig. 3. 17. and A. B. Fig. 18. and 19.

45. A Parallelogram whoſe Sides are all equal and Angles right, is called a Square, as ABCD. Fig. 17.

46. A Parallelogram whoſe oppoſite Sides are equal and Angles right, is called a Rectangle or an Oblong, as ABCD. Fig. 3.

47. A Rhombus is a Parallelogram of equal Sides, and has its Angles oblique, as A. Fig. 18. and is an inclined Square.

Plate I.

48. A Rhomboides is a Parallelogram whoſe oppoſite Sides are equal and Angles oblique; as B. Fig. 19. and may be conceived as an inclined Rectangle.

49. Any quadrilateral Figure that is not a Parallelogram is called a Trapezium. Plate 7. Fig. 3.

50. Figures which conſiſt of more than four Sides are called Polygons; if the Sides are all equal to each other they are called regular Polygons. They ſometimes are named from the Number of their Sides, as a five-ſided Figure is called a Pentagon, one of ſix Sides a Hexagon, *&c.* but if their Sides are not equal to each other, then they are called irregular Polygons, as an irregular Pentagon, Hexagon, *&c.*

51. Four Quantities are ſaid to be in Proportion when the Product of the Extremes is equal to that of the Means: Thus if A multiplied by D, be equal to B multiplied by C. Then A is ſaid to be to B as C is to D.

POSTULATES or PETITIONS.

1. That a right Line may be drawn from any one given Point to another.

2. That a right Line may be produced or continued at Pleaſure.

3. That from any Center and with any Radius the Circumference of a Circle may be deſcribed.

4. It is alſo required, that the Equality of Lines and Angles to others given, be granted as poſſible:

That it is poſſible for one right Line to be perpendicular to another, at a given Point or Diſtance; and that every Magnitude has its half, third, fourth, *&c.* Part.

Note, Though theſe Poſtulates are not always quoted, the Reader will eaſily perceive where and in what Senſe they are to be underſtood.

AXIOMS or Self-evident TRUTHS.

1. Things that are equal to one and the ſame Thing, are equal to each other.

2. Every Whole is greater than its Part.

3. Every Whole is equal to all its Parts taken together.

4. If to equal Things, equal Things be added, the Wholes will be equal.

5. If from equal Things, equal Things be deducted, the Remainders will be equal.

6. If to or from unequal Things, equal Things be added or taken, the Sums or Remainders will be unequal.

7. All right Angles are equal to one another.

8. If two right Lines not parallel, be produced towards their neareſt Diſtance, they will interſect each other.

9. Things which mutually agree with each other are equal.

NOTES.

A Theorem is a Propoſition, wherein ſomething is propoſed to be demonſtrated.

A Problem is a Propoſition, wherein ſomething is to be done or effected.

A Lemma is ſome Demonſtration, previous and neceſſary, to render what follows the more eaſy.

A Corollary is a conſequent Truth, deduced from a foregoing Demonſtration.

A Scholium, is when Remarks or Obſervations are made upon ſomething going before.

The Signification of SIGNS.

The Sign =, denotes the Quantities between which it ſtands to be equal.

The Sign +, denotes the Quantity it precedes to be added.

The Sign —, denotes that Quantity which it precedes to be ſubſtracted.

The Sign X, denotes the Quantities between them to be multiplied into each other.

To denote that four Quantities, A, B, C, D, are proportional, they are uſually wrote thus, A : B : : C : D; and read thus A is to B, ſo is C to D; but when three Quantities A, B, C, are proportional the middle Quantity is repeated, and they are wrote A : B : : B : C.

GEOMETRICAL THEOREMS.

THEOREM I.

Plate I.

IF a right Line falls on another, as AB, or EB, does on CD, (Fig. 20.) it either makes with it two right Angles, or two Angles equal to two right Angles.

1. If AB be perpendicular to CD, then (by Def. 11.) the Angles CBA, and ABD, will be each a right Angle.

2. But if EB fall ſlantwiſe on CD, then are the Angles DBE + EBC = DBE + EBA (= DBA) + ABC, or to two right Angles. Q. E. D.

Corollary I. Whence if any Number of right Lines were drawn from one Point, on the ſame Side of a right Line; all the Angles made by theſe Lines will be equal to two right Angles.

2. And all the Angles which can be made about a Point, will be equal to four right Angles.

THEO.

THEO. II.

Plate I.

If one right Line croſſes another, (as AC does BD) the oppoſite Angles made by thoſe Lines, will be equal to each other. That is AEB to CED and BEC to *AED Fig.* 21.

By Theorem I. BEC + CED = 2 right Angles,
and CED + DEA = 2 right Angles.

Therefore (by Axiom 1.) BEC+CED = CED+DEA: take CED from both, and there remains BEC = DEA. (by Axiom 5.) Q. E. D.

After the ſame Manner CED + AED = 2 right Angles; and AED+AEB = 2 right Angles; wherefore taking AED from both, there remains CED = AEB. Q. E. D.

THEO. III.

If a right Line croſſes two Parallels, as GH does AB and CD. (Fig. 22.) then

1. *Their external Angles are equal to each other, that is GEB = CFH.*

2. *The alternate Angles will be equal, that is AEF = EFD and BEF = CFE.*

3. *The external Angle will be equal to the internal and oppoſite one on the ſame Side, that is GEB = EFD and AEG = CFE.*

Plate I.

4. *And the Sum of the internal Angles on the ſame Side are equal to two right Angles; that is BEF+DFE are equal to two right Angles, and AEF+CFE are equal to two right Angles.*

1. Since AB is parallel to CD, they may be conſidered as one broad Line, croſſed by another Line, as GH; (then by the laſt Theo.) GEB=CFH, and AEG=HFD.

2. Alſo GEB = AEF, and CFH=EFD; but GEB = CFH (by Part 1. of this Theo.) therefore AEF=EFD. The ſame Way we prove FEB=EFC.

3. AEF=EFD; (by the laſt Part of this Theo.) but AEF=GEB (by Theo. 2.) Therefore GEB = EFD. The ſame Way we prove AEG=CFE.

4. For ſince GEB = EFD to both add FEB, then (by Axiom 4.) GEB+FEB=EFD+FEB, but GEB+FEB, are equal to two right Angles (by Theo. 1.) Therefore EFD+FEB are equal to two right Angles: After the ſame Manner we prove that AEF+CFE are equal to 2 Right Angles. Q. E. D.

THEO. IV.

In any Triangle ABC, one of its Legs as BC being produced towards D, it will make the external Angle ACD equal to the two internal oppoſite Angles taken together. Viz. to B and A. Fig. 23.

Thro'

Plate I.

Thro' C, let CE be drawn parallel to AB; then ſince BD cuts the two parallel Lines, BA, CE; the Angle ECD = B, (by Part 3 of the laſt Theo.) and again, ſince AC cuts the ſame Parallels the Angle ACE = A (by Part 2. of the laſt.) Therefore ECD+ACE = ACD = B+A. Q. E. D.

THEO. V.

In any Triangle ABC, all the three Angles taken together are equal to two right Angles, viz. A+B+ACB = 2 right Angles. Fig. 23.

Produce BC to any Diſtance as D, then (by the laſt) ACD = B+A; to both add ACB; then ACD + ACB = A + B + ACB: But ACD + ACB = 2 right Angles (by Theo. 1.); therefore the three Angles A+B + ACB = 2 right Angles. Q. E. D.

Cor. 1. Hence if one Angle of a Triangle be known, the Sum of the other two is alſo known: For ſince the three Angles of every Triangle contain two right ones, or 180 Degrees, therefore 180 — the given Angle will be equal to the Sum of the other two; or 180 — the Sum of two given Angles, gives the other one.

Cor. 2. In every right-angled Triangle, the two acute Angles are = 90 Degrees, or to one right Angle: Therefore 90 — one acute Angle, gives the other.

THEO.

Plate I.

THEO. VI.

If any two Triangles, ABC, DEF, there be two Sides AB, AC, in the one, ſeverally equal to DE, DF in the other, and the Angle A contained between the two Sides in the one, equal to D in the other; then the remaining Angles of the one, will be ſeverally equal to thoſe of the other, viz. B = E and C = F: And the Baſe of the one BC, will be equal to EF that of the other. Fig. 24.

If the Triangle ABC be ſuppoſed to be laid on the Triangle DEF, ſo as to make the Points A and B coincide with D and E, which they will do, becauſe AB = DE (by the Hypotheſis); and ſince the Angle A = D, the Line AC will fall along DF, and inaſmuch as they are ſuppoſed equal, C will fall in F; ſeeing therefore the three Points of one coincide with thoſe of the other Triangle, they are manifeſtly equal to each other; therefore the Angle B = E and C = F, and BC = EF. Q. E. D.

LEMMA.

If two Sides of a Triangle abc be equal to each other, that is ac = cb; the Angles which are oppoſite to thoſe equal Sides will alſo be equal to each other, viz. a = b. Fig. 11.

For let the Triangle abc be divided into two Triangles acd, dcb, by making the Angle acd = dcb (by Poſtulate 4.) then becauſe ac = bc, and cd, common, (by the laſt) the Triangle adc = dcb; and therefore the Angle a = b. Q. E. D.

Cor.

Plate I.

Cor. Hence if from any Point in a Perpendicular which biſects a given Line, there be drawn right Lines to the Extremities of the given one, they with it will form an Iſoſceles Triangle.

THEO. VII.

The Angle BCD at the Center of a Circle ABED, is double the Angle BAD at the Circumference. Fig. 25.

Through the Point A, and the Center C, draw the Line ACE: Then the Angle ECD = CAD + CDA; (by Theo. 4) but ſince AC = CD being Radii of the ſame Circle, it is plain (by the preceding Lemma) that the Angles ſubtended by them will be alſo equal, and that their Sum is double to either of them, that is DAC+ADC is double to CAD, and therefore ECD is double to CAD; after the ſame Manner BCE is double to CAB, wherefore, BCE+ECD or BCD is double to BAC+CAD or to BAD. Q. E. D.

Cor. 1. Hence an Angle at the Circumference is meaſured by half the Arc it ſubtends or ſtands on.

Cor. 2. Hence all Angles at the Circumference of a Circle which ſtands on the ſame Chord as AB, are equal to each other, for they are all meaſured by half the Arc they ſtand on, *viz.* by half the Arc AB. Fig. 26.

Cor.

Plate I.

Cor. 3 Hence an Angle in a Segment greater than a Semi-circle is leſs than a right Angle; thus ADB is meaſured by half the Arc AB, but as the Arc AB is leſs than a Semicircle, therefore half the Arc AB, or the Angle ADB is leſs than half a Semicircle, and conſequently leſs than a Right Angle. Fig. 26.

Cor. 4. An Angle in a Segment leſs than a Semicircle is greater than a right Angle, for ſince the Arc AEC is greater than a Semicircle, it's half, which is the Meaſure of the Angle ABC, muſt be greater than half a Semicircle, that is greater than a right Angle. Fig. 27.

Cor. 5. An Angle in a Semicircle is a right Angle, for the Meaſure of the Angle ABD is half of a Semicircle AED, and therefore a right Angle.

THEO. VIII.

If from the Center C of a Circle ABE, there be let fall the Perpendicular CD on the Chord AB, it will biſect it in the Point D. Fig. 29.

Let the Lines CA and CB be drawn from the Center to the Extremities of the Chord, then ſince CA=CB, the Angle CAB=CBA (by the Lemma). But the Triangles ADC, BDC are right angled ones, ſince the Line CD is a Perpendicular, and ſo the Angle ACD=DCB; (by Cor. 2. Theo. 5.) then have we AC, CD, and the Angle ACD in one Triangle; ſeverally equal to CB, CD, and the Angle BCD in the other: Therefore (by Theo. 6) AD=DB. Q. E. D.

So

Plate I.

Cor. Hence it follows, that any Line biſecting a Chord at right Angles is a Diameter; for a Line drawn from the Center perpendicular to a Chord biſects that Chord at Right Angles; therefore converſely a Line biſecting a Chord at right Angles muſt paſs thro' the Center, and conſequently be a Diameter.

THEO. IX.

If from the Center of a Circle ABE there be drawn a Perpendicular CD on the Chord AB, and produced till it meets the Circle in F, that Line CF will biſect the Arc AB in the Point F. Fig. 29.

Let the Lines AF and BF be drawn, then in the Triangles ADF, BDF; AD=BD (by the laſt); DF is common, and the Angle ADF=BDF being both right, for CD or DF is a Perpendicular. Therefore (by Theo. 6.) AF=FB; but in the ſame Circle equal Lines are Chords of equal Arcs, ſince they meaſure them (by Def. 19.); whence the Arc AF=FB, and ſo AFB is biſected in F, by the Line CF.

Cor. Hence the Sine of an Arc is half the Chord of twice that Arc. For AD is the Sine of the Arc AF, (by Def. 22.) AF is half the Arc, and AD half the Chord AB (by Theo. 8.) Therefore the Cor. is plain.

THEO. X.

Plate I.

In any Triangle ABD, the half of each Side is the Sine of the oppoſite Angle. Fig. 30.

Let the Circle ADB be drawn thro' the Points A, B, D; then the Angle DAB is meaſured by half the Arc BKD, (by Cor. 1. Theo. 7.) *viz.* the Chord of BK is the Meaſure of the Angle BAD; therefore (by Cor. to the laſt) BE the half of BD is the Sine of BAD: The ſame Way may be proved, that half of AD is the Sine of ABD, and the half of AB the Sine of ADB. Q. E. D.

THEO. XI.

If a right Line GH cuts two other right Lines AB; CD, ſo as to make the alternate Angles AEF, EFD equal to each other, then the Lines AB and CD will be parallel. Fig. 22.

If it be denied that AB is parallel to CD, let IK be parallel to it; then IEF = (EFD) = AEF (by Part 2. Theo. 3.) a greater to a leſs which is abſurd, whence IK is not parallel; and the like we can prove of all other Lines but AB; therefore AB is parallel to CD. Q. E. D.

THEO. XII.

If two equal and parallel Lines AB, CD, be joined by two other Lines AD, BC, thoſe ſhall be alſo equal and parallel. Fig. 3.

Let

Plate I.

Let the Diameter or Diagonal BD be drawn, and we will have the two Triangles ABD, CDB; whereof AB in one is = to CD in the other, DB common to both, and the Angle ABD = CDB (by Part 2. Theo. 3.); therefore (by Theo. 6.) AD = CB, and the Angle CBD = ADB, and thence the Lines AD and BC are parallel, by the preceding Theorem.

Cor. 1. Hence the Quadrilateral Figure ABCD is a Parallelogram, and the Diagonal BD biſects the ſame, inaſmuch as the Triangle ABD = BDC, as now proved.

Cor. 2. Hence alſo the Triangle ADB on the ſame Baſe AB, and between the ſame Parallel with the Parallelogram ABCD, is half the Parallelogram.

Cor. 3. It is hence alſo plain, that the oppoſite Sides of a Parallelogram are equal; for it has been proved that ABCD being a Parallelogram, AB will be = CD and AD = BC.

THEO. XIII.

All Parallelograms on the ſame or equal Baſes and between the ſame Parallels are equal to one another, that is if BD = GH, and the Lines BH and AF parallel, then the Parallelogram ABDC = BDFE = EFHG. Fig. 31.

For AC (= DB) = EF (by Cor. the laſt); to both add CE, then AE = CF. In the Triangles ABE, CDF; AB = CD and AE = CF and the Angle BAE = DCF (by Part 3. Theo. 3.); therefore the Triangle ABE = CDF, (by Theo. 6.), let the Triangle CKE be taken from both, and we will

Plate I.

have the Trapezium ABKC = KDFE; to each of theſe add the Triangle BKD, then the Parallelogram ABCD = BDEF; in like Manner we may prove the Parallelogram EFHG = BDEF. Wherefore ABDC = BDEF = EFHG. Q. E. D.

Cor. Hence it is plain that Triangles on the ſame or equal Baſes, and between the ſame parallels, are equal, ſeeing (by Cor. 2. Theo. 12.) they are the Halves of their reſpective Parallelograms.

THEO. XIV.

In every right-angled Triangle, ABC, the Square of the Hypothenuſe or longeſt Side, BC, or BCMH, is equal to the Sum or the Squares made on the other two Sides AB and AC, that is to ABDE and ACGF. (Fig. 32.)

Thro' A draw AKL perpendicular to the Hypothenuſe BC, joyn AH, AM, DC and BG; in the Triangles BDC, ABH, BD = BA being Sides of the ſame Square, and alſo BC = BH, and the included Angle DBC = ABH, (for DBA = CBH being both right, to both add ABC, then DBC = ABH) therefore the Triangle DBC = ABH (by Theo. 6,) but the Triangle DBC is half of the Square ABDE (by Cor. 2. Theo. 12) and the Triangle ABH is half the Parallelogram BKLH (by the ſame); therefore half the Square ABDE is equal to half the Parallelogram BKLH, and the Square ABDE equal to the Parallelogram BKLH. The ſame Way it my be proved, that the Square ACGF, is equal to the Parallelogram KCLM. So ABDE + ACGF the Sum of the Squares,

Plate I.

Squares, = BKLH+KCML, the Sum of the two Parallelograms or Square BCMH; therefore the Sum of the Squares on AB and AC is equal to the Square on BC. Q. E. D.

Cor. 1. Hence the Hypothenuſe of a right-angled Triangle may be found by having the Legs; thus, the Square Root of the Sum of the Squares of the Baſe and Perpendicular, will be the Hypothenuſe.

Cor. 2. Having the Hypothenuſe and one Leg given to find the other; the Square Root of the Difference of the Squares of the Hypothenuſe and given Leg, will be the required Leg.

THEO. XV.

In all Circles the Chord of 60 *Degrees is always equal in Length to the Radius.*

Thus in the Circle AEBD, if the Arc AEB be an Arc of 60 *Degrees, and the Chord AB be drawn; then AB = CB = AC. (Fig.* 33.*)*

In the Triangle ABC, the Angle ACB is 60 Degrees being meaſured by the Arc AEB; therefore the Sum of the other two Angles is 120 Degrees (by Cor. 1. Theo. 5.) but ſince AC = CB, the Angle CAB = CBA (by Lemma preceding Theo. 7.) conſequently each of them will be 60 the half of 120 Degrees, and the three Angles will be equal to one another, as well as the three Sides: Wherefore AB = BC = AC. Q. E. D.

Cor.

Plate I.

Cor. Hence the Radius, from whence the Lines on any Scale is formed, is the Chord of 60 Degrees on the Line of Chords.

THEO. XVI.

If in two Triangles ABC, abc, all the Angles of one, be each reſpectively equal to all the Angles of the other, that is A = a, B = b, C = c: Then the Legs oppoſite to the equal Angles will be proportional, viz.

AB : ab : : AC : ac Fig. 34.
AB : ab : : BC : bc
and AC : ac : : BC : bc

For the Triangles being inſcribed in two Circles, it is plain ſince the Angle A = a, the Arc BDC = bdc, and conſequently the Chord BC is to bc, as the Radius of the Circle ABC is to the Radius of the Circle abc; (for the greater the Radius is, the greater is the Circle deſcribed by that Radius; and conſequently the greater any particular Arc of that Circle is, ſo the Chord, Sine, Tangent, &c. of that Arc will be alſo greater. Therefore in general, the Chord, Sine, Tangent, &c. of any Arc is proportional to the Radius of the Circle); the ſame Way the Chord AB is to the Chord ab, in the ſame Proportion. So AB : ab : : BC : bc; the ſame Way the reſt may be proved to be proportional.

THEO. XVII.

If from a Point A without a Circle DBCE there be drawn two Lines ADE, ABC, each of them cutting the Circle in two Points; the Product of one whole

Plate I.

whole Line into its external Part, viz. AC into AB, will be equal to that of the other Line into its external Part, viz. AE into AD. Fig. 35.

Let the Lines DC, BE be drawn, in the two Triangles ABE, ADC; the Angle AEB = ACD, (by Cor. 2. Theo. 7) the Angle A is common, and (by Cor. 1. Theo. 5.) the Angle ADC = ABE; therefore the Triangles ABE, ADC, are mutually equiangular, and conſequently, (by the laſt) AC : AE : : AD : AB; wherefore AC multipled by AB, will be equal to AE multiplied by AD. Q. E. D.

THEO. XVIII.

Plate II. Fig. 1.

Triangles ABC, BCD, and Parallelograms ABCF, and BDEC, having the ſame Altitude, have the ſame Proportion between themſelves as their Baſes AB, and BD.

Let any aliquot Part of AB be taken, which will alſo meaſure BD: Suppoſe that to be Ag which will be contained twice in AB, and three Times in BD, the Parts Ag, gB, Bh, hi, and iD being all equal, and let the Lines gC, hC, and iC, be drawn: Then (by Cor. to Theo. 13.) all the ſmall Triangles AgC, gCB, BCh, *&c.* will be equal to each other; and will be as many as the Parts into which their Baſes were divided: Therefore it will be, as the Sum of the Parts in one Baſe, is to the Sum of thoſe in the other, ſo will be the Sum of ſmall Triangles in the firſt, to the Sum of the ſmall Triangles in the ſecond Triangle; that is AB : BD : : ABC : BDC.

Whence

Plate II.

Whence alſo the Parallelograms ABCF, and BDEC being (by Cor. 2. Theo. 12) the Doubles of the Triangles, are likewiſe as their Baſes. Q. E. D.

Note. Wherever there are ſeveral Quantities connected with the Sign : : The Concluſion is always drawn from the firſt two, and laſt two Proportionals.

THEO. XIX.

Triangles ABC, DEF, ſtanding upon equal Baſes AB and DE, are to each other as their Altitudes CG and FH. Fig. 2.

Let BI be perpendicular to AB and equal to CG, in which let KB = FH, and let AI and AK be drawn.

The Triangle AIB = (ACB by Cor. to Theo. 13) and AKB = DEF; but (by Theo. 18.) BI : BK : : ABI : ABK. That is CG : FH : : ABC : DEF. Q. E. D.

THEO. XX.

If a right Line BE be drawn parallel to one Side of a Triangle ACD, it will cut the two other Sides proportionally, viz. *AB : BC : : AE : ED. Fig.* 3.

Draw CE and BD; the Triangles BEC and EBD being on the ſame Baſe BE and under the ſame Parallel CD, will be equal (by Cor. to Theo. 13.) therefore (by Theo. 18.) AB : BC : : (BEA ; BEC or BEA : BED) : : AE : ED. Q. E. D.

Cor.

Plate II.

Cor. 1. Hence alſo AC : AB : : AD : AE: For, AC : AB : : (AEC : AEB : : ABD : AEB) : : AD : AE.

Cor. 2. It alſo appears that a right Line, which divides two Sides of a Triangle proportionally, muſt be parallel to the remaining Side.

Cor. 3. Hence alſo Theo. 16. is manifeſt; ſince the Sides of the Triangles ABE, ACD being equiangular, are proportional.

THEO. XXI.

If two Triangles ABC, ADE, have one Angle in BAC, equal to one Angle DAE, and the Sides about the equal Angles proportional, that is AB : AD : : AC : AE, then the Triangles will be mutually equiangular. Fig. 4.

In AB take Ad = AD, and let de, be parallel to BC, meeting AC in e.

Becauſe (by the firſt Cor. to the foregoing Theo.) AB : Ad : : (AD) AC : Ae, and (by the Hypotheſis or what is given in the Theorem) AB : AD : AC : AE; therefore Ae = AE ſeeing AC bears the ſame Proportion to each; and (by Theo. 6.) the Triangle Ade = ADE, therefore the Angle Ade = D and Aed = E, but ſince ed and BC are parallel (by Part 3. Theo. 3.) Ade = B, and Aed = C, therefore B = D and C = E. Q. E. D.

Plate II.

THEO. XXII.

Equiangular Triangles ABC, DEF, are to one another in a Duplicate Proportion of their Homologous or like Sides; or as the Squares AK, and DM of their Homologous Sides. Fig. 5.

Let the Perpendiculars CG and FH be drawn, as well as the Diagonals BI and EL.

The Perpendiculars make the Triangles ACG and DFH equiangular, and therefore ſimilar (by Theo. 16.) for becauſe the Angle CAG = FDH and the right Angle AGC = DHF, the remaining Angle ACG = DFH, (by Cor. 2. Theo. 5.)

Therefore GC : FH : : (AC : DF : :) AB : DE or which is the ſame Thing GC : AB : : FH : DE, for FH multiplied by AB = AB multiplied by FH.

By Theo. 19 ABC : ABI : : (CG : AI, or AB as beſore : : FH DE, or DL : :) DFE : DLE, therefore ABC : ABI : : DFE : DLE, or ABC : AK : : DFE : DM, for AK is double the Triangle ABI, and DM double the Triangle DEF, by Cor. 2. Theo. 11. Q. E. D.

THEO. XXIII.

Like Polygons ABCDE, abcde, are in a Duplicate Proportion to that of the Sides AB, ab, which are between the equal Angles A and B, and a and b, or as the Squares of the Sides AB, ab. Fig. 6.

Draw AD, AC, ad, ac.

By

Plate II.

By the Hypotheſis AB : ab : : BC : bc, and thereby alſo the Angle B = b ; therefore (by Theo. 21.) BAC = bac ; and ACB = acb ; in like Manner EAD = ead, and EDA = eda. If therefore from the equal Angles A, and a, we take the equal ones EAD+BAC = ead+bac the remaining Angle DAC = dac, and if from the equal Angles D and d, EDA = eda be taken, we will have ADC = adc : and in like Manner if from C and c be taken BCA = bca we ſhall have ACD = acd ; and ſo the reſpective Angles in every Triangle will be equal to thoſe in the other.

By Theo. 22. ABC : abc : : the Square of AC to the Square of ac, and alſo ADC : adc : : the Square of AC, to the Square of ac ; therefore from Equality of Proportions ABC : abc : : ADC : adc, in like Manner we may ſhew that ADC : adc : EAD : ead ; therefore it will be as one Antecedent, is to one Conſequent ; ſo are all the Antecedents, to all the Conſequents. That is ABC : abc as the Sum of the three Triangles in the firſt Polygon, is to the Sum of thoſe in the laſt. Or ABC will be to abc, as Polygon to Polygon.

The Proportion of ABC to abc (by the foregoing Theo.) is as the Square of AB is to the Square of ab, but the Proportion of Polygon to Polygon is as ABC to abc as now ſhewn : therefore the Proportion of Polygon to Polygon is as the Square of AB, to the Square of ab.

Plate I.

THEO. XXIV.

Let DHB be a Quadrant of a Circle deſcribed by the Radius CB : HB an Arc of it, and DH its Complement; HL or FC the Sine, FH or CL its Co-Sine; BK its Tangent, DI its Co-tangent; CK its Secant, and CI its Co-ſecant. Fig. 8.

1. The Co-ſine of an Arc, is to the Sine, as Radius, is to the Tangent.

2. Radius is to the Tangent of an Arc, as the Co-ſine of it is to the Sine.

3. The Sine of an Arc is to its Co-ſine, as Radius to its Co-tangent.

4. Or Radius is to the Co-tangent of an Arc, as its Sine to its Co ſine.

5. The Co tangent of an Arc is to Radius, as Radius to the Tangent.

6. the Co-Sine of an Arc is to Radius, as Radius is to the Secant.

7. The Sine of an Arc is to Radius, as the Tangent is to the Secant.

The Triangles CLH, and CBK being ſimilar, (by Theo. 16.)

1. CL : LH : : CB : BK.
2. Or, CB : BK : : CL : LH.

The

Plate I.

The Triangles CFH, and CDI, being Similar.

3. CF (or LH) : FH : : CD : DI.

4. CD : DI : : CF, (or LH) : FH.

The Triangles CDI and CBK are ſimilar; for the Angle CID = KCB, being alternate ones (by Part 2. Theo. 3.) the Lines CB and DI being parallel: The Angle CDI = CBK being both right, and conſequently the Angle DCI = CKB, wherefore

5. DI : CD : : CB : BK.

And again, making Uſe of the ſimilar Triangles CLH, and CBK.

6. CL : CB : : CH : CK.

7. HL : CH : : BK : CK.

GEOMETRICAL PROBLEMS.

PROB. I.

Plate II.

TO make a Triangle of three given right Lines BO, LB, LO, of which any two muſt be greater than the third. Fig. 7.

Lay BL from B to L, from B with the Line BO, deſcribe an Arc, and from L with LO deſcribe another Arc; from O, the Interſection Point of thoſe Arcs draw BO, and OL, and BOL is the Triangle required.

This is manifeſt from the Conſtruction.

PROB. II.

At a Point B in a given Right Line BC, to make an Angle equal to a given Angle A. Fig. 8.

Draw any right Line ED to form a Triangle as EAD, take BF = AD, and upon BF make the Triangle BFG whoſe Side BG = AE, and GF = ED (by the laſt) then alſo the Angle B = A; if we ſuppoſe one Triangle be laid on the other, the Sides will mutually agree with each other, and therefore be equal, for if we conſider theſe two Triangles are made of the ſame given three Lines, they are manifeſtly one and the ſame Triangle.

Otherwiſe

Plate II.

Upon the Centers A and B, at any Diſtance, let two Arcs, DE, FG, be deſcribed; make the Arc FG = DE, and thro' B and G draw the Line BG, and it's done.

For ſince the Chords ED, GF, are equal, the Angles A and B are alſo equal, as before (by Def. 19.)

PROB. III.

To biſect or divide into two equal Parts, any given right-lined Angle, BAC. Fig. 9.

In the Lines AB and AC, from the Point A ſet off equal Diſtances AE = AD, then, with any Diſtance, more than the half of DE, deſcribe two Arcs to cut each other in ſome Point F; and the right-line AF, joining the Points A, and F, will biſect the given Angle BAC.

For if DF and FE be drawn, the Triangles ADF, AEF, are equilateral to each other, *viz.* AD = AE, DF = FE, and AF common, wherefore DAF = EAF, as before.

PROB. IV.

To biſect a right line, AB. Fig. 10.

With any Diſtance, more than half the Line, from A and B, deſcribe two Circles CFD, CGD, cutting each other in the Points C and D; draw CD, interſecting AB in E, then AE = EB.

For

Plate II.

For if AC, AD, BC, BD, be drawn; the Triangles ACD, BCD, will be mutually equilateral, and conſequently the Angle ACE = BCE: Therefore the Triangles ACE, BCE, having AC = BC, CE common, and the Angle ACE = BCE; (by Theo. 6.) the Baſe AE = the Baſe BE.

Cor. Hence it is manifeſt that CD not only biſects AB, but is perpendicular to it. (by Def. 11.)

PROB. V.

On a given Point A, in a right Line EF, to erect a Perpendicular. Fig. 11.

From the Point A lay off on each Side, the equal Diſtances, AC, AD; and from C and D, as Centers, with any Interval greater than AC or AD, deſcribe two Arcs interſecting each other in B; from A to B draw the Line AB, and it will be the Perpendicular required.

For, let CB, and BD be drawn; then the Triangles CAB, DAB, will be mutually equilateral and equiangular, ſo CAB = DAB, a right Angle, (by Def. 11.)

PROB. VI.

To raiſe a Perpendicular on the End B of a right Line AB. Fig. 12.

From any Point D not in the Line AB, with the Diſtance from D to B, let a Circle be deſcribed cutting AB in E; draw from E thro' D the right Line EDC, cutting the Periphery in C, and join CB; and that is the Perpendicular required.

Plate II.

EBC being a Semicircle, the Angle EBC will be a right Angle, (by Cor. 5. Theo. 7.)

PROB. VII.

From a given Point A, to let fall a Perpendicular upon a given right Line BC. Fig. 13.

From any Point D, in the given Line, take the Diſtance to the given Point A, and with it deſcribe a Circle AGE, make GE=AG, join the Points A and E, by the Line AFE, and AF will be the Perpendicular required.

Let DA, DE, be drawn; the Angle ADF=FDE, DA=DE, being Radii of the ſame Circle, and DF common; therefore (by Theo. 6.) the Angle DFA =DFE, and FA a Perpendicular. (by Def. 11.)

PROB. VIII.

Thro' a given Point A, to draw a right Line AB, parallel to a given right Line CD. Fig. 14.

From the Point A, to any Point, F, in the Line CD, draw the Line AF; with the interval FA, and one Foot in F deſcribe the Arc AE, and with the like Interval and one Foot in A, deſcribe the Arc BF, making BF=AE; thro' A and B draw the Line AB, and it will be parallel to CD.

By Prob. 2. The Angle BAF=AFE, and by Theo. 11. BA and CD are parallel.

Plate II.

PROB. IX.

Upon a given Line AB, to deſcribe a Square ABCD. Plate I. Fig. 17.

Make BC perpendicular and equal to AB; and from A and C with the Line AB, or BC, let two Arcs be deſcribed cutting each other in D; from whence to A and C, let the Lines AD, DC be drawn; ſo is ABCD the Square required.

For all the Sides are equal by Conſtruction; therefore the Triangles ADC, and BAC are mutually equilateral and equiangular, and ABCD is an equilateral Parallelogram, whoſe Angles are right. For B being right, D is alſo right, and DAC, DCA, BAC, ACB, each half a right (by Lemma preceding Theo. 7. and Cor. 2. Theo. 5.) whence DAB and BCD will each be a right Angle, and (by Def. 44.) ABCD is a Square.

SHOLIUM.

By the ſame Method a Rectangle or Oblong may be deſcribed, the Sides thereof being given.

PROB. X.

To divide a given right Line AB, into any propoſed Number of equal Parts. Fig. 15.

Draw the infinite right Line AP, making any Angle with AB, alſo draw BQ parallel to AP, in each of which let there be taken as many equal Parts

Plate II.

Parts AM, MN, *&c.* Bo, on, *&c.* as you would have AB divided into; then draw Mm, Nn, *&c.* interſecting AB in E, F, *&c.* and it's done.

For MN, and mn, being equal and parallel, FN will be parallel to EN; and in the ſame Manner GO to FN (by Theo. 12.) therefore AM, MN, NO being all equal by Conſtruction, it is plain (from Theo. 20.) that AE, EF, FG, *&c.* will likewiſe be equal.

PROB. XI.

To find a third proportional to two given right Lines, A and B. Fig. 16.

Draw two infinite blank Lines CE, CD, any wiſe to make an Angle. Lay the Line A from C to F, and the Line B from C to G, and draw the Line FG; lay again the Line A from C to H, and thro' H, draw HI parallel to FG, (by Prob. 8.) ſo is CI the third proportional required.

For by Cor. 1. Theo. 20. CG : CH : : CF : CI.

Or B : A : : A : CI.

PROB. XII.

Three right Lines A, B, C, given to find a fourth proportional. (Fig. 17.)

Having

Plate II.

Having made an Angle DEF any wiſe, by two infinite blank right Lines, ED, EF, as before; lay the Line A from E to G, the Line B from E to I, and drawn the Line IG, lay the Line C from E to H, and (by Prob. 8.) draw HK parallel thereto, ſo will EK be the fourth Proportional required.

For by Cor. 1. Theo. 20. EG : EI : : EH : EK.
Or A : B : : C : EK.

PROB. XIII.

Plate III.

Two right Lines, A and B, given, to find a Mean proportional. (Fig. 1.)

Draw an infinite blank Line, as AF, on which lay the Line A, from A to B, and the Line B, from B to C, on the Point B, which is the joining of the Lines A and B, erect a Perpendicular BD, (by Prob. 5.) biſect AC in E (by Prob. 4.) and deſcribe the Semicircle ADC; and from the Point D, where its Periphery cuts the Perpendicular BD, draw the Line BD, and that will be the Mean Proportional required.

For if the Lines AD, DC, be drawn, the Angle ADC is a right Angle, (by Cor. 5. Theo. 7.) being an Angle in a Semicircle.

The Angles ABD, DBC, are right ones, (by Def. 11.) the Line BD being a Perpendicular; wherefore the Triangles ABD, DBC, are ſimilar, thus the Angle ABD = DBC, being both right, the Angle DAC is the Complement of BDA to a right Angle, (by Cor. 2. Theo 5.) and is therefore equal to BDC, the Angle ADC being a right Angle as before; conſequently (by Cor. 1. Theo. 5.) the Angle ADB = DCE, wherefore (by Theo. 16.)

AB : BD : : BD : BC.
Or A ; BD : : BD ; B.

PROB. XIV.

To divide a right Line AB, in the Point E, ſo that AE ſhall have the ſame Proportion to AB, as two given Lines C and D have. (Fig. 2.)

Draw an infinite blank Line, AF, to the Extremity of the Line AB, to make with it any Angle; lay the Line C from A to C, and D from A to D, and join the Points B and D, by the Line BD, thro' C draw CE parallel to BD (by Prob. 8.) ſo is E the Point of Diviſion.

For by Cor. 1. Theo. 20. AC ; AD : : AE : AB.
Or C : D ; : AE ; AB.

PROB. XV.

To deſcribe a Circle about a Triangle ABC (or which is the ſame Thing) thro' any three Points, A, B, C, which are not ſituate in a right Line. (Fig. 3.)

By Prob. 4. Biſect the Line AC by the Perpendicular DE, and alſo CB, by the Perpendicular FG, the Point of Interſection H of theſe Perpendiculars is the Center of the Circle required, from which take the Diſtance to any of the three Points A, B, C, and deſcribe the Circle ABC and it is done.

For

Plate III.

For by Cor. to Theo. 8. The Lines DE and FG muſt each paſs thro' the Center, therefore their Point of Interſection H, muſt be the Center.

SCHOLIUM.

By this Method the Center of a Circle may be found, by having only a Segment of it given.

PROB. XVI.

To make an Angle of any Number of Degrees, at the Point A of the Line AB, ſuppoſe of 45 *Degrees.* (*Fig.* 4.)

From a Scale of Chords take 60 Degrees, for 60 is equal to the Radius, (by Cor. Theo. 15.) and with that Diſtance from A as a Center, deſcribe a Circle from the Line AB; take 45 Degrees, the Quantity of the given Angle, from the ſame Scale of Chords, and lay it on that Circle from a to b, thro' A and b, draw the Line AbC; and the Angle A, will be an Angle of 45 Degrees as required.

If the given Angle were more than 90, take its half (or divide it into any two Parts leſs than 90) and lay them after each other on the Arc which is deſcribed with the Chord of 60 Degrees; thro' the Extremity of which and the Center, let a Line be drawn, and that will form the Angle required, with the given Line.

Plate III.

PROB. XVII.

To meaſure a given Angle ABC. (*Fig.* 5.)

If the Lines which include the Angle be not as long as the Chord of 60 on your Scale, produce them to that or a greater Length, and between them ſo produced, with the Chord of 60 from B, deſcribe the Arc ed: which Diſtance ed meaſured on the ſame Line of Chords, gives the Quantity of the Angle BAC 48 Degrees as required; this is plain from Def. 19.

PROB. XVIII.

To make a Triangle BCE equal to a given Quadrilateral Figure ABCD. Fig. 6.

Draw the Diagonal AC, and Parallel to it (by Prob. 8.) DE, meeting AB produced in E; then draw CE, and ECB will be the Triangle required.

For the Triangles ADC, AEC being upon the ſame Baſe AC, and under the ſame Parallel ED, (by Cor. to Theo. 13) will be equal, therefore if ABC be added to each, then ABCD = BEC.

PROB. XIX.

To make a Triangle DHF, equal to a given five-ſided Figure ABCDE. Fig. 7.

Draw DA and DB, and alſo EH and CF parallel to them, (by Prob. 8.) meeting AB produced in H and

Plate III.

H and F; then draw DH, DF, and the Triangle HDF is the one required.

For the Triangle DEA = DHA, and DBC = DFB (by Cor. to Theo. 13.) therefore by adding theſe Equations DEA + DBC = DHA + DFB, if to each of theſe ADB be added; then DEA + ADB + DBC = ABCDE = (DHA + ABD + DFB) = DHF.

P R O B. XX.

To project the Lines of Chords, Sines, Tangents, and Secants to any Radius. Fig. 8.

On the Line AB, let a ſemicircle ADB be deſcribed; let CD be drawn perpendicular to the Center C, and the Tangent BE perpendicular to the End of the Diameter; let the Quadrants AD, DB be each divided into 9 equal Parts, every of which will be 10 Degrees; if then from the Center C, Lines be drawn thro' 10, 20, 30, 40, *&c.* the Diviſions of the Quadrant BD, and continued to BE, we will there have the Tangents of 10, 20, 30, 40, *&c.* and the Secants C 10, C 20, C 30, *&c.* are transferred to the Line CD, produced by deſcribing the Arcs 10, 10: 20, 20: 30: 30, *&c.* If from 10, 20, 30, *&c.* the Diviſions of the Quadrant BD there be let fall Perpendiculars, let theſe be transferred to the Radius CD, and we will have the Sines of 10, 20, 30, *&c.* and if from A we deſcribe the Arcs 10, 10: 20, 20: 30: 30, *&c.* from every Diviſion of the Arc AD; we will have a Line of Chords. The ſame Way we may have the Sine Tangent, *&c* to every ſingle Degree on the Quadrant, by ſubdividing every of the 9 former Diviſions into 10 equal Parts. By this Method the Sines, Tangents, *&c.* may be drawn to any Radius; and if after they be transferr'd to Lines on a Rule, we ſhall have the Scales of Sines, Tangents, *&c.* ready for uſe.

Concerning Scales of equal Parts.

If an Inch be divided into any aſſigned Number of equal Parts, and if theſe Parts be continued on in a right Line, and if the laſt of them be ſubdivided into 10 equal Parts, and thence if the firſt Diviſions be numbered with 1, 2, 3, 4, *&c.* as far as the Ruler upon which they are transferred will admit, the Scale is compleated.

Theſe Numbers 1, 2, 3, 4, *&c.* uſually ſtand for 10, 20, 30, 40, *&c.* and every one of the Subdiviſions is called 1: But if the Numbers 1, 2, 3, 4, *&c.* be called 100, 200, 300, 400, *&c.* then every one of the Subdiviſions will be 10, and the Units muſt be gueſſed at.

On one Side of moſt ſurveying Scales, there are Lines, or Scales, marked at the End with 50, 45, 40, 35, 30, 25, 15, 10, and ſometimes with other Numbers; theſe are Scales of ſo many Parts to an Inch, (whether of Feet, Yards, Perches, or Miles,) as the reſpective Number at the End of each expreſſes; but in the Surveying Way, they are counted to be ſo many Perches to an Inch, and ſometimes ſo many Feet to an Inch.

On the contrary Side there are two Scales, one of 10, and the other of 20; or one of 100, and the other of 200; or one of 1000, and the other of 2000 Parts to an Inch, diagonally divided; (a View of the Scale will make all eaſy:) The firſt of theſe *Surveyors* call a Scale of 10, and the other a Scale of 20 Perches to an Inch; and are thus counted: Every large Diviſion is 10, every one of the Subdiviſions is 1, and every one downwards is one tenth of a Perch; or ſometimes thus, every large Diviſion is called 100, every Subdiviſion 10, and every one downwards 1: Or again, frequently by *Navigators*, every large Diviſion is called 1000, every Subdiviſion 100, every one downwards 10, and the tenth Part of the Diſtance between the Lines 1.

Hence it is eaſy to meaſure the Length of any Line, knowing the Scale by which it was laid down; and on the contrary, to ſet off any given Diſtance, from any Scale.

Of

Of LOGARITHMS.

IF to a Series of Numbers in *Geometrical Progreſſion*, whoſe common *Ratio* is 10, and firſt Term 1; we annex another Series of Numbers in *Arithmetical Progreſſion*, whoſe firſt Term is 0, and common Difference 1: Theſe latter Numbers will be the Logarithms of the former.

Numbers.	Logarithms.
1	0.00000
10	1.00000
100	2.00000
1000	3.00000
10000	4.00000, *&c.*

If ſeveral Geometrical Means be taken, and the like Number of Arithmetical ones, to the correſponding Numbers, the latter will be the Logarithms of the former.

The Nature therefore of Logarithms is ſuch, that Addition of them anſwers to the Multiplication of their correſponding Numbers; and Subtraction to Diviſion: That is, when two Numbers propoſed are to be multiplied into each other, if we take the Logarithms anſwering to thoſe Numbers, and add them together, the Sum will be the Logarithm anſwering to the Number, which is the Product of the two propoſed Numbers.

Again, when one Number is propoſed to be divided by another; if from the Logarithm of the Dividend, we ſubſtract the Logarithm of the Diviſor, the Remainder ſhall be the Logarithm of the Quotient.

Moſt Tables of Logarithms contain the Logarithms of all Numbers from 1 to 10000, the Column mark'd at the Top N, is that in which you muſt find your Number; in the ſame Line with which, in the adjacent Column, is the Logarithm of that Number.

EXAMPLE.

Required the Logarithm of 365.

Anſwer, 2.56229.

And though moſt Tables of Logarithms run but to 10000, yet by them the Log. of any Number not exceeding 10,000,000 may be found, and on the contrary, the Number to any ſuch Logarithm, thus,

1. Find the Log. of the firſt four Figures of the given Number.

2. Take that Log. from the Log. of the Number next following, and note their Difference.

3. Multiply that Difference by the remaining Figures of the given Number; and from the Product, cut off as many Figures as remain in the given Number, or as the given Number is more than four (counting from the Right to the Left) as in Decimals.

4. The whole Number in the Product, added to the firſt Log. is the Log. required; but the firſt Figure, which is called the Index, or Characteriſtick, muſt be changed; and always be one leſs than the Number of Figures in the Logarithm.

EXAMPLE I.

Required the Logarithm of the Number 3567894.

The Log. of 3567, which are the four firſt Figures is 3.55230

The Log. of the following or next Number, *viz.* 3568 is 3.55242

Their Difference, - - - - 12
Mult. by the remaining Fig. *viz.* .894

Cut off 3 Figures, becauſe 894 is 3 Figures, and the Product is, - - - - 10.728
To which add the firſt Log. 3.55230

Their Sum is 3.55240

But becauſe the given Number conſiſts of 7 Figures, the Index muſt be one leſs which is 6; ſo the above Index 3, muſt be changed to 6, and we have 6.55240 the Log. of 3567894 required.

EXAMPLE II.

Required the Log. of the Number 125607.
The Log. of 1256 is 3.09899
The next Log. following is 3.09934

Their

Their Difference is - -	35
Multiply by .07 the remaining Figures	.07
Product - - -	2.45
To which add the firſt Log.	3.09899
Their Sum is - -	3.09901

Becauſe the given Number conſiſts of 6 Places, change the laſt Index to 5, which is one leſs than the Places in the given Number; and you have 5.09901, the Log. of 125607 required.

Becauſe any Number conſiſting of both Integers and Decimals, is equal to the Quotient of the whole conſidered as an Integer, divided by the Denominator of the Decimal Part; and ſince by the Nature of Logarithms, Subſtraction in them anſwers the Quotient of other Numbers; therefore it follows, that when a Number is given conſiſting of Integers, and Decimals, we can find its Log. thus: Find the Log. of the whole conſidered as one Integer; then from that, take the Log. of the Denominator of the Decimal Part; or (which is the ſame thing) from the Index of the Log. of the whole conſidered as an Integer, ſubſtract a Number leſs by one, than the Number of Places in the Denominator of the Fraction, and the Remainder will be the Log. required; or the Index of the Log. muſt be 1 leſs, than the Number of Figures in the Integer to which the Decimal is annex'd.

EXAMPLE I.

What is the Log. of the Number 36.5?

Find the Log. of 365, which is 2.56229; then becauſe 10 is the Denominator of the Decimal Part of the propoſed Number, and 1.00000 it's Log. therefore from 2.56229, take 1.00000, and there remains 1.56229 the Log. required.

Or, becauſe the whole Number conſiſts of two Figures, the Index of the Log. muſt be one leſs, and is therefore 1.56229, as before.

EXAMPLE II.

What are the Logs of 6543, 654.3, 65.43, 6.543, .6543, .06543 and .006543?

6543	3.81578
654.3	2.81578
65.43	1.81578
6.543	0.81578
.6543	$\bar{9}.81578$
.06543	$\bar{8}.81578$
.006543	$\bar{7}.81578$

For the Log. of a Decimal Fraction is the ſame as that of an Integer; only the Index is Negative, and is ſo much leſs than 0. as the Place of the Decimal is removed from Unity; and thoſe Indices may be diſtinguiſhed from abſolute ones, by ſetting a Negative Sign over them, as above.

To find the Number of a given Logarithm.

Look for the given Log. amongſt the Logs from 1000 to 10000 (not regarding the Index or firſt Figure) and if you find the exact Log. you want, you have in the Margin the required Number. But if the Index of the given Log. be leſs than 3, cut off from the Number found, as many Figures as it is leſs; and the Figures ſo cut off will be Decimals, and the others Integers. Or if the firſt Figure, or Index, be greater than 3, add as many Cyphers to the Number found as it is more, and you have the Number required.

EXAMPLES.

Find the Numbers correſpondent to the following Logarithms.

Given Logarithms.		Numbers.
5.55230	Anſwer	356700.
4.55230	—	35670.
3.55230	—	3567.
2.55230	—	356.7
1.55230	—	35.67
0.55230	—	3.567
9.55230	—	.3567
8.55230	—	.03567
7.55230	—	.003567, &c.

But if the exact Log. cannot be found in the Table, and the Number of Figures required, exceed four, then

1. Find as before (not regarding the Index) the Log. anſwering to the firſt four Figures, but leſs than the given Log.

2. Take that from the given one, and if the Remainder do not conſiſt of two Figures, prefix a Cypher to it; and after theſe two Figures annex three Cyphers, ſo will you have five Figures for a Dividend.

3. Divide that by the Difference between the Log. found, and the next following, and if your Quotient do not conſiſt of three Figures, prefix a Cypher or Cyphers to make it; which three Figures place after the firſt four found.

Then obſerve the Index of the given Log. which ſhews how many Figures muſt be Integers, and how many Decimals; for the Number of Integers is one more, than the given Index as before.

EXAMPLE I.

1. Required the Number of the Log. 4·55241

The neareſt Log. which is leſs is 3·55230 its Number is 3567.

The Difference of theſe with three Cyphers is for a Dividend 11000

The Log. found 3·55230
The next Log. 3·55242

Their Difference will give for a Diviſor 12

12)11000(916 Quotient
108
20
12
80

Which Quotient place after the firſt four Figures found, and you have 3567916; and becauſe the Index is 4, the Number will be 35679.16 required.

2. Required the Number anſwering to the Log. 5·09901.

The neareſt Log. to which is 3·09899 its No. 1256

Dividend 02000

Log. found 3.09899
Next Log. 3.09934

```
35 Diviſor 35)02000(57
                175
                ---
                250
                245
                ---
                  5
```

Becauſe the Quotient conſiſts of but two Figures, prefix a Cypher to it to make it three, and it is 057: which annex'd to the firſt four found, is 1256057; and becauſe the Index of the given Log. is 5, its Number will be 125605.7.

From what has been ſaid on this Head, the following Problems may eaſily be ſolv'd by Logarithms, *viz.*

PROB. I.

Multiply 134, by 25.6

To Log. of 134	2.12710
Add the Log. of 25.6	1.40824
Sum	3.53534

The Number anſwering to which Sum, *viz.* 3430, is nearly the Product of 134 by 25.6 and is the Anſwer.

Again, Multiply 234 by 36.

To Log. of 234 2.36922
Add the Log of 36 1.55630

Sum 3.92552 its Number

is 8424 required.

PROB. II.

What's the Quotient of 828 by 23?

From the Log. of 828 2.91803
Take the Log. of 23 1.36173

Difference 1.55630 its Number

36, the Quotient required.

Again, what's the Quotient of 30550 by 47?

From the Log. of 30550 4.48501
Take the Log. of 47 1.67210

2.81291 its Numb

is 650 the Quotient required.

PROB. III.

Three Numbers in a direct Proportion given, to find a Fourth.

From the Sum of the Logarithms of the second and third Numbers; deduct the Logarithm of the first, the Remainder will be the Logarithm of the fourth required.

EXAMPLE I.

Let the three propoſed Numbers be 36, 48, 66, to find a fourth proportional.

To Log. of 48 1.68124
Add Log. of 66 1.81954

Product 3.50078
Take Log. of 36 1.55630

1.94448 the Number is 88, the fourth required.

Again, let three Numbers be 240, 1440, 1230, to find a fourth proportional.

To the Log. of 1440 3.15836
Add the Log. of 1230 3.08991

From the Product 6.24827
Take the Log of 240 2.38021

3.86806 its Number 7380 the 4th required.

PROB. IV.

To find the Square of any given Number.

Multiply the given Numbers Logarithm by 2, and the Product is the Logarithm of its Square.

EXAM-

EXAMPLE.

Required the Square of 36.

Log. of 36 1.55630
Multiply by 2

3.11260 it's Number 1296 the Square required.

PROB. V.

To extract the Square Root of any given Number.

Take half of the Logarithm of the Number, and that is the Logarithm of it's Square Root.

EXAMPLE.

Required the Square Root of 1296.

Log. of 1296 3.11261

It's half is 1.55630 it's Number is 36, the Square Root of the Number required.

By the Manner of Projecting the Lines of Chords, Sines, Tangents and Secants (being Prob. 20 of Geometry) it is evident, that if the Radius be ſuppoſed any Number of equal Parts (as 1000 or 10000, *&c.*) the Sine, Tangent, *&c.* of every Arc, muſt conſiſt of ſome Number of thoſe equal Parts; and by computing them in Parts of the Radius,

dius, we have Tables of Sines, Tangents, *&c.* to every Arc of the Quadrant called natural Sines, Tangents, *&c.* and the Logarithms of theſe, give us Tables of logarithmic Sines, Tangents, *&c.* and ſuch are uſually bound up with Logarithms of Numbers.

In which you may obſerve, that each Page is divided into 8 Columns, the firſt and laſt of which are Minutes, and the intermediate ones contain the Sines, Tangents and Secants, the upper and lower Columns contain Degrees, the Column of the Minutes on the left Hand of each Page, anſwers to the Degrees in the Top Column; and the Sines, Tangents and Secants belonging to thoſe Degreee and Minutes, are in the Columns mark'd at the Top with the Words Sine, Tangent, and Secant; the Column of Minutes on the right-Hand of each Page anſwers to the Degrees in the Bottom of the Page; and the Sines, Tangents and Secants, anſwering to thoſe Degrees and Minutes, are in the Columns, mark'd at the Bottom with the Word Sine, Tangent, Secant; the Degrees in the Top Column beginning at 0, proceed to 44 where they end; and thoſe at the Bottom of the Page begin at 89, and proceed to 45 in a decreaſing Series; the Degrees in the different Columns being the Complement of each other. From what has been ſaid, we may eaſily find the Sine, Tangent or Secant of any Arc from the Tables, by looking for the given Number of Degrees, at the Head or Foot of the Page, according as they are leſs or greater than 45, and in the proper Side Column for the odd Minutes, if there be any; then below or above the Word Sine, Tangent or Secant, and on the ſame Line with the Minutes, we ſhall have that which was required.

EXAMPLE I.

Required the Sine of 36 Degrees, 40 Minutes.

Look at the Head of the Page for 36 Degrees, and in the Side Column on the left Hand, for 40 Minutes; then below the Word Sine, on the ſame Line with 40, we find 9.77609; which is that required.

EXAMPLE II.

Required the Tangent of 54 Degrees 30 Minutes.

Look at the Foot of the Page, (becauſe the propoſed Degrees are more than 45.) for 54 Degrees, and in the right Hand Column for 30 Minutes; then in the Column mark'd Tangent at it's Bottom, and on the ſame Line with the 30 Minutes, in the Side Column, we find 10.14673, which is the Log-tangent required.

The Reverſe of this, *viz.* The Logarithm of a Sine, Tangent, or Secant, being given, to find the Arc belonging to it, is performed by only looking in the proper Column for the neareſt Logarithm to that propoſed, and the Degrees and Minutes anſwering thereto, are thoſe required.

We will now ſhew how any Sine, Tangent, or Secant may be had, tho' the Figures in the Tables were defaced, miſprinted, or obliterated.

PROB. I.

To find the Tangent which is defaced, by the Sine and Co-ſine.

The Co-ſine taken from the Sine added to 90, or Radius, which is 10.00000, the Remainder is the Tangent. (by Part 1. Theo. 24.)

EXAMPLE.

1. Suppoſe the Tangent of 41°. 20', was defaced, but the Sine and Co-ſine of it viſible.

From the Sine of 41°. 20' + 10.00000, or Radius, — — —	19.81983
Take the Co-ſine of 41°. 20'	9.87557
The Rem. is the Tan. of 41°. 20' req. *viz.*	9.94426

2. To find a Sine which is miſprinted, by Help of the Co-ſine and Tangent.

From the Sum of the Tangent and Co-ſine, take 10.00000, or Radius, or (which is the ſame Thing) cut off the firſt Figure of the Index, the Remainder is the Sine required, (by Part 2. Theo. 24.)

EXAMPLE.

Suppoſe the Sine of 46°. 50' was defaced, but the Tangent and Co-ſine viſible.

To the Tangent of 46°. 50'	10.02781
Add the Co-ſine of 46°. 50'	9.83513
Their Sum is the Sine of 46°. 50' req. *viz.*	9.86294

The Co-tangent and Co-ſine of any Arc may be had by the ſame Method; the Complement of any Degree, being only it's Reſidue from 90, or a

Quadrant, as before obſerved, (by Theo. 24. Part 3 and 4.)

3. To find a Tangent by the Help of a Co-tangent only.

From twice the Radius, which is 20.00000; take the Co-tangent, the Remainder is the Tangent, (by Theo. 24. Part 5.)

EXAMPLE.

Required the Tangent of 29°. 50′ being defaced, as alſo the Sine and Co-ſine defaced, by the Co-tangent only.

From twice the Radius,	20.00000
Take the Co-tangent of 29°. 50′	10.24148
The Rem. is the Tang. of 29°. 50′ req.	9.75852

4. To find the Secant, by the Help of a Co-ſine; which may be found of great Uſe, when a Table of Sines and Tangents can only be had.

From twice the Radius, which is 20.00000 take the Co-ſine, and the Remainder will be the Secant, (by Theo. 24. Part 6.)

EXAMPLE.

Required the Secant of 57°. 20′ by the Help of the Co-ſine only.

From the double Radius,	20.00000
Take the Co-ſine of 57°. 20′	9.73219
The Rem. is the Secant of 57°. 20′ req.	10.26781

5. To find a Secant by the Help of the Sine and Tangent.

From the Tangent added to Radius, take the Sine, the Remainder will be the Secant, (by Theo. 24. Part 7.)

EXAMPLE.

Required the Secant of 57°. 20′ by Help of the Sine and Tangent.

From the Tan. of 57°. 20′ + 10.00000 the Radius,	20.19303
Take the Sine of 57°. 20′	9.92522
The Rem. is the Secant of 57°. 20′ req.	10.26781

The Secants in theſe Tables might have been omitted, becauſe all Proportions in which they are concerned may be wrought by Sines and Tangents only, as ſhall be ſhewn in the ſeveral Caſes of Plane Trigonometry; and are here only inſerted that all the various Methods of reſolving Triangles may be ſhewn.

SECT. II.

Containing Plane Trigonometry, right-angled and oblique; with it's Application in determining the Meaſures of inacceſſible Heights and Diſtances.

Plane Trigonometry

IS the Science of meaſuring the Sides and Angles of Plane Triangles. It is divided into two Parts, *viz.* into *Rectangular* and *Oblique angular Trigonometry*, becauſe every Triangle is either right-angled or oblique; and therefore we ſhall begin with

RECTANGULAR TRIGONOMETRY.

Plate V. Fig. 1.

1. In every right-angled Plane Triangle ABC, if the Hypothenuſe AC be made the Radius, and with it a Circle, or an Arc of one, be deſcribed from each End; it is plain (from Def. 22.) that BC is the Sine of the Angle A, and AB is the Sine of the Angle C; that is, the Legs are the Sines of their oppoſite Angles.

Plate V.

2. If one Leg AB be made the Radius, and with it, on the Point A, an Arc be deſcribed; then BC is the Tangent, and AC is the Secant of the Angle A, by Def. 24 and 25. Fig. 2.

3. If BC be made the Radius, and an Arc be deſcribed with it on the Point C; then is AB the Tangent, and AC is the Secant of the Angle C, as before. Fig. 3.

Becauſe the Sine, Tangent, or Secant of any given Arc in one Circle, is to the Sine, Tangent, or Secant of a like Arc (or to one of the like Number of Degrees,) in another Circle; as the Radius of the one is to the Radius of the other; therefore the Sine, Tangent, or Secant of any Arc is proportional to the Sine, Tangent, or Secant of a like Arc, as the Radius of the given Arc is to 10.00000, the Radius from whence the Logarithmic Sines, Tangents and Secants, in moſt Tables, are calculated, i. e.

If AC be made the Radius, the Sines of the Angles A and C, deſcribed by the Radius AC, will be proportional to the Sines of the like Arcs, or Angles in the Circle, that the Tables now mentioned were calculated for. So if BC was required, having the Angles and AB given, it will be Fig. 1.

As S.C : AB : : S.A : BC.

i. e. As the Sine of the Angle C in the Tables, is to the Length of AB; (or Sine of the Angle C, in a Circle whoſe Radius is AC); ſo is the Sine of the Angle A in the Tables, to the Length of BC. (or Sine of the ſame Angle, in the Circle, whoſe Radius is AC.)

In like Manner, the Tangents and Secants, repreſented by making either Leg the Radius; will be proportional to the Tangents and Secants of a like Arc, as the Radius of the given Arc is to 10.00000, the Radius of the Tables aforeſaid.

Hence it is plain, that if the Name of each Side of the Triangle be placed thereon, a Proportion will ariſe to anſwer the ſame End as before: Thus if AC be made the Radius, let the Word Radius be wrote thereon; and as BC, and AB, are the Sines of their oppoſite Angles; upon the firſt let S.A, or Sine of the Angle A, and on the other, let S. C, or Sine of the Angle C, be wrote: Then if a Side be required, it may be obtained by this Proportion, *viz.*

As the Word on the Side given,
  is to the Side given,
So is the Word on the Side required
  to the Side required.

Thus if the Angles A and C, and the Hypothenuſe AC were given, to find the Legs; the Proportions will be

1 R : AC : : S. A : BC. Fig. 1.
That is, as Radius is to AC; ſo is the Sine of the Angle A, to BC. And,
2. R : AC : : S. C : AB.
That is, as Radius is to AC; ſo is the Sine of the Angle C, to AB.

Plate V.

But when an Angle is required, we uſe this Proportion, *viz.*

As the Side that is made the Radius,
  is to Radius;
So is the other given Side,
  to the Word upon it.

Thus, if the Legs were given, to find the Angle A, and if AB be made the Radius, it will be,

AB : R : : BC : T. A. Fig. 2.

That is, as AB, is to Radius; ſo is BC, to the Tangent of the Angle A.

After the ſame Manner, the Sides or Angles of all right-angled plane Triangles may be found, from their proper Data.

We here, in Plate 4, give all the Proportions requiſite for the Solution of the ſix Caſes in Rectangular Trigonometry; making every Side poſſible the Radius.

In the following Triangles, this Mark — in an Angle, denotes it to be known, or the Quantity of Degrees it contains to be given; and this Mark ı on a Side, denotes it's Length to be given, in Feet, Yards, Perches, or Miles, *&c.* and this Mark °, either in an Angle or on a Side, denotes the Angle or Side required.

From

Plate V.

From theſe Proportions it may be obſerved; that, to find a Side, when the Angles and one Side are given, any Side may be made the Radius: And to find an Angle one of the given Sides muſt be made the Radius. So that in the 1ſt, 2d, and 3d Caſes, any Side, as well required as given, may be made the Radius; and in the firſt Statings of the 4th, 5th and 6th Caſes, a given Side only is made the Radius.

RECTANGULAR TRIGONOMETRY.

CASE I.

THE *Angles and Hypothenuſe given, to find the Baſe and Perpendicular. Fig.* 4.

In the right-angled Triangle ABC, ſuppoſe the Angle A 46°. 30', and conſequently the Angle C 43°. 30' (by Cor. 2. Theo. 5.); and AC 250 Parts, (as Feet, Yards, Miles, *&c.*) required the Legs AB, and BC.

Geometrically.

Make an Angle of 46°. 30', in blank Lines, (by Prob. 16. Sect. 1.) as CAB; lay 250, which is the given Hypothenuſe, from a Scale of equal Parts, from A to C; from C, let fall the Perpendicular BC, (by Prob. 7. Sect. 1.) and that will conſtitute the Triangle ABC. Meaſure the Lines BC, and AB, from the ſame Scale of equal Parts, that AC was taken from; and you have the Anſwer.

Plate V.

By Calculation.

1. *Making AC the Radius,* the required Sides are found by theſe Proportions, as in Plate 4, Caſe 1.

R : AC : : SA : B.C.
R : AC : : SC : A.B.

i. e.	As Radius,	90°	10.00000
	is to AC,	250	2.39794
	So is the Sine of A	46°. 30′	9.86056
	to BC,	181. 4	2.25850

As Radius,	90°	10.00000
is to AC,	250	2.39794
So is the Sine of C	43°. 30′	9.83781
to AB,	172. 1	2.23575

If from the Sum of the ſecond and third Logs, that of the firſt be taken, the Remainder will be the Log. of the fourth; the Number anſwering to which will be the Thing required; but when the firſt Log. is Radius, or 10.00000, reject the firſt Figure of the Sum of the other two Logs; (which is the ſame Thing as to ſubſtract 10.00000); and that will be the Log. of the Thing required.

2. *Making AB the Radius.*

Secant A : AC : : R : AB.
Secant A : AC : : T. A : BC.

i. e.

i. e.	As the Secant of A	46°. 30′	10.16219
	is to AC	250	2.39794
	So is Radius	90°	10.00000
			12.39794
	to AB	172 . 1	2.23575

As the Secant of A	46°. 30′	10.16219
is to AC	250	2.39794
So is the Tangent of A	46°. 30′	10.02275
		12.42069
to BC	181 . 4	2.25850

3. *Making BC the Radius.*

Sec. C : AC : : R : BC.
Sec. C : AC : : T . C : AB.

i. e.	As the Secant of C	43°. 30′	10.13944
	is to AC	250	2.39794
	So is Radius	90°	10.00000
			12.39794
	to BC	181 . 4	2.25850

As the Secant of C	43°. 30′	10.13944
is to AC	250	2.39794
So is the Tangent of C	43°. 30′	9.97725
		12.37519
to AB	172 . 1	2.23575

Or having found one Leg, the other may be obtained by Cor. 2. Theo. 14. Sect. 1.

By Gunter's *Scale.*

On this Scale, there are Lines of Numbers, Sines, and Tan. as well as Lines of Sine and Tan. Rumbs, Verſed Sines, Meridional Parts, and Equal Parts: But the three firſt Lines are ſufficient for our preſent Purpoſe.

The Diviſions on theſe reſpective Lines, are the Logarithms of Numbers, Sines, and Tangents, taken from a Scale of equal Parts, and applied on the Lines of the Scale.

The firſt and third Terms in the foregoing Proportions being of a like Nature, and thoſe of the ſecond and fourth being alſo like to each other; and the Proportions being direct ones, it follows; that if the third Term be greater, or leſs than the firſt, the fourth Term will be alſo greater or leſs than the ſecond: Therefore the Extent in your Compaſſes from the firſt, to the third Term, will reach from the ſecond to the fourth.

Thus, to extend the firſt of the foregoing Proportions:

1. Extend from 90°, to 46°. 30', on the Line of Sines; that Diſtance will reach from 250 on the Line of Numbers, to 181, for BC.

2. Extend from 90° to 43°. 30' on the Line of Sines; that Diſtance will reach from 250 on the Line of Numbers, to 172, for AB.

If the firſt Extent be from a greater, to a leſs Number; when you apply one Point of the Compaſſes to the ſecond Term, the other muſt be turned to a leſs; and the contrary.

By Def. 22. Sect. 1. The Sine of 90° is equal to the Radius; and the Tangent of 45° is alſo equal to the Radius; becauſe if one Angle of a right-angled Triangle be 45°, the other will be alſo 45°; and thence (by the Lemma preceding Theo. 7. Sect. 1.) the Tangent of 45° is equal to the Radius: For this Reaſon the Line of Numbers of 10.00000, the Sine of 90°, and Tangent of 45°, being all equal, terminate at the ſame End of the Scale; where there are ſmall Braſs Centers, uſually placed, to preſerve the Scale.

It was ſaid before, that the Tangents ended at 45°; but becauſe the Logarithms of Tangents more than 45°, muſt paſs off the Scale; ſuch Diſtances therefore as exceed 45°, are ſet backwards from 45, and numbered 50, 60, 70, *&c.*

There is no Line of Secants on the Scale; for every Thing requiſite, can be performed without them.

Thus the two firſt Statings of this Caſe, anſwer the Queſtion without a Secant: The like will be alſo made evident in all the following Caſes.

CASE 2.

The Baſe and Angles given; to find the Perpendicular and Hypothenuſe.

Plate V. Fig. 5.

In the Triangle ABC, there is the Angle A 42°. 20', and of Courſe the Angle C 47°. 40', (by Cor. 2. Theo. 5.) and the Leg. AB 190, given; to find BC and AC.

Geometrically.

Make the Angle CAB (by Prob. 16. Sect. 1.) in blank Lines as before. From a Scale of equal Parts lay 190 from A to B; on the Point B erect a Perpendicular BC (by Prob. 5. Sect. 1.) the Point where this cuts the other blank Line of the Angle, will be C; ſo is the Triangle ABC conſtructed: Let AC, and BC be meaſured, from the ſame Scale of equal Parts, that AB was taken from, and you have the Anſwer.

By Calculation.

1. *Making AC the Radius.*

S. C : AB : : R : AC.
S. C : AB : : S. A. BC.

i. e.	As the Sine of C	47°. 40′	9.86879
	is to AB	190	2.27875
	So is Radius	90°	10.00000
			12.27875
	to AC	257	2.40996

As the Sine of C	47°. 40′	9.86879
is to AB	190	2.27875
So is the Sine of A	42°. 20′	9.82830
		12.10705
to BC	173.1	2.23826

2. *Making AB the Radius.*

R : AB : : T. A : BC
R : AB : : Sec. A : AC

i. e.	As Radius	90°	10.00000
	is to AB	190	2.27875
	So is the Tangent of A	42°. 20′	9.95952
	to BC	173.1	2.23827

As Radius,	90°	10.00000
is to AB,	190	2.27875
So is the Secant of A	42°. 20′	10.13121
to AC	257	2.40996

3. *Making BC the Radius.*

T. C : AB : : Sec. C : AC.
T. C : AB : : R : BC.

i. e. As the Tangent of C	47°. 40′	10.04048
is to AB,	190	2 27875
So is the Secant of C	47°. 40′	10.17170
		12.45045
to AC,	257	2.40997

As the Tangent of C	47°. 40′	10.04048
is to AB,	190	2.27875
So is Radius,	90°	10.00000
		12.27875
to BC,	173. 1	2.23827

Or having found one of the required Sides, the other may be obtained, by one or the other of the Cors. to Theo. 14. Sect. 1.

By Gunter's Scale.

1. When AC is made the Radius.

Extend from 47° 40′, to 90°, on the Line of

Sines; that Diſtance will reach from 190 to 257, on the Line of Numbers, for AC.

2. When AB is made the Radius, the firſt Stating is thus performed.

Extend from 45° on the Tangents, (for the Tangent of 45° is equal to the Radius, or to the Sine of 90°, as before) to 42°. 20'; that Extent will reach from 190, on the Line of Numbers, to 173, for BC.

3. When BC is made the Radius, the ſecond Stating is thus performed.

Extend from 47°. 40', on the Line of Tangents, to 45°, or Radius; that Extent will reach from 190 to 173, on the Line of Numbers, for BC; for the Tangent of 47°. 40', is more than the Radius; therefore the fourth Number muſt be leſs than the ſecond, as before.

The two firſt Statings of this Caſe, anſwer the Queſtion without a Secant.

CASE 3.

The Angles and Perpendicular given; to find the Baſe and Hypothenuſe.

Plate V. Fig. 6.

In the Triangle ABC, there is the Angle A 40°, and conſequently the Angle C 50°, with BC 172, given; to find AC and AB.

Geometrically.

Make an Angle CAB of 40° in blank Lines; (by Prob. 16. Sect. 1.) with BC 172, from a Line of equal Parts, draw the popped Lines EF parallel to AB (by Prob. 8. Sect. 1.) the lower Line of the Angle, and from the Point where it cuts the other Line in C, let fall a Perpendicular BC (by Prob. 7. Sect. 1.) and the Triangle is conſtructed: The Meaſures of AC, and AB, from the ſame Scale that BC was taken, will anſwer the Queſtion.

What has been ſaid in the two foregoing Caſes, is ſufficient to render the Operations in this, both by *Calculation* and *Gunter*'s *Scale*, ſo obvious, that it is needleſs to inſert them; however, for the ſake of the Learner, we give for

Anſwer, AC 264. 5, and AB 202. 6.

CASE 4.

The Baſe and Hypothenuſe given; to find the Angles and Perpendicular.

Plate V. Fig. 7.

In the Triangle ABC, there is given, AB 300, and AC 500: The Angles A and C, and the Perpendicular BC, are required.

Geometrically.

From a Scale of equal Parts, lay 300 from A, to B; on B erect an infinite blank perpendicular Line, with AC 500, from the ſame Scale, and one Foot of the Compaſſes in A, croſs the perpendicular

Line in C; and the Triangle is conſtructed.

By Prob. 17. Sect. 1. Meaſure the Angle A, and let BC be meaſured from the ſame Scale of equal Parts that AC and AB were taken from; and you have the Anſwer.

By Calculation.

1. *Making AC the Radius.*

AC : R : : AB : S. C.
R : AC : : S. A : BC.

i. e. As AC	500	2.69897
is to Radius,	90°	10.00000
So is AB	300	2.47712
		12.47712
to the Sine of C	36°. 52′	9.77815

By Cor. 2. Theo. 5. 90°—36°. 52′=53°. 08′ the Angle A.

As Radius,	90°	10.00000
is to AC,	500	2.69897
So is the Sine of A	53°. 08′	9.90301
to BC,	400	2.60198

2. *Making AB the Radius.*

AB : R : : AC : Sec. A.
R : AB : : T. A : BC.

i. e.

i. e. As AB,	300	2.47712
is to Radius,	90°	10.00000
So is AC,	500	2.69897
		12.69897
to the Secant of A,	53°. 08'	10.22185

As Radius,	90°	10.00000
is to AB,	300	2.47712
So is the Tangent of A	53°. 08'	10.12499
to BC,	400	2.60211

Or BC may be found from Cor. 1. Theo. 14. Sect. 1.

By Gunter's Scale.

1. Making AC the Radius.

Extend from 500 to 300, on the Line of Numbers; that Extent will reach from 90°, on the Line of Sines, to 36°. 52', for the Angle C.

Again. Extend from 90° to 53°. 08', on the Line of Sines, that Extent will reach from 500 to 400, on the Line of Numbers, for BC.

2. Making AC the Radius, the ſecond Stating is thus performed.

Extend from Radius, or the Tangent of 45°, to 53°. 08', that Extent will reach from 300 to 400, for BC.

CASE V.

The Perpendicular and Hypothenuſe given, to find the Angles and Baſe.

Plate V. Fig. 8.

In the Triangle ABC, there is BC 306, and AC 370, given; to find the Angles A and C, and the Baſe AB.

Geometrically.

Draw a blank Line, from any Point, in which at B, erect a Perpendicular, on which lay BC 306, from a Scale of equal Parts: From the ſame Scale, with AC 370, in the Compaſſes, croſs the firſt drawn blank Line in A, and you have the Triangle ABC conſtructed.

Meaſure the Angle A (by Prob. 17. Sect. 1.); and alſo AB, from the ſame Scale of equal Parts the other Sides were taken from, and you have the Anſwer.

The Operations by Calculation, the Square Root, and *Gunter*'s Scale are here omitted, as they have been heretofore fully explained: The Statings, or Proportions muſt alſo be obvious, from what has already been ſaid.

Anſwer. The Angle A 55°. 48'; therefore the Angle C 34°. 12', and AB 208.

CASE 6.

The Baſe and Perpendicular given; to find the Angles and Hypothenuſe.

Plate V. Fig. 9.

In the Triangle ABC, there is AB 225, and BC 272, given; to find the Angles A and C, and the Hypothenuſe AC.

Geometrically.

Draw a blank Line, on which lay AB 225, from a Scale of equal Parts; at B erect a Perpendicular, on which lay BC 272, from the ſame Scale; join A and C, and the Triangle is conſtructed.

As before, let the Angle A, and the Hypothenuſe AC be meaſured; and you have the Anſwer.

By Calculation.

1. *Making AB the Radius.*

AB : R : : BC : T. A.
R : AB : : Sec. A : AC.

2. *Making BC the Radius.*

BC : R : : AB : T. C.
R. : BC : : Sec. C : AC.

By Calculation; the Anſwer from the foregoing Proportions is eaſily obtained, as before.

But becauſe AC, by either of the ſaid Proportions, is found by Means of a Secant; and ſince there is no Line of Secants on *Gunter*'s Scale; after having found the Angles, as before, let us ſuppoſe AC the Radius, and then

1. S. A : BC : : R. : AC.
or 2. S. C : AB : : R. : AC.

Theſe Proportions may be eaſily reſolved, either by *Calculation*, or *Gunter*'s *Scale*, as before; and thus the Hypothenuſe AC may be found without a Secant.

From the two given Legs, the Hypothenuſe may be eaſily obtained, from Cor. 1. Theo. 14. Sect. 1.

Thus, to the Square of AB = 50625
Add the Square of BC = 73984

```
      124609(353 = AC
       9
    65)346
       325
   703)2109
       2109
```

From what has been ſaid on Logarithms, it is plain,

1. That half the Logarithm of the Sum of the Squares of the two Sides, will be the Logarithm of the Hypothenuſe. Thus

The Sum of Squares, as before, is 124609; it's Log. is 5.09554, the half of which is 2.54777;

and the correſponding Number to this, in the Tables, will be 353, for AC.

2. And that half of the Logarithm of the Difference of the Squares of AC and AB, or of AC and BC, will be the Logarithm of BC, or of AB.

The following Examples are inſerted for the Uſe of the Learner.

1. Given,	the Angle C AC	64°. 40′ 3876	AB BC	required.
2. Given,	the Angle C AB	47°. 20′ 17	AC BC	required.
3. Given,	the Angle C BC	28°. 30′ 27187	AB AC	required.
4. Given,	AB AC	2 3	the Angles and BC	required.
5. Given,	BC AC	17 21.6	the Angles and AB	required.
6. Given,	AB BC	2871.64 3176.2	the Angles and AC	required.

The Anſwers are omitted, that the Learner may reſolve them himſelf by the foregoing Methods; by which Means he will find and ſee more diſtinctly their mutual Agreements; and become more expert, and the better acquainted with the Subject.

OBLIQUE ANGULAR PLANE TRIGONOMETRY.

BEFORE we proceed to the Solution of the four Caſes of Oblique-Angular Triangles, it is neceſſary to premiſe the following Theorems.

THEO. I.

Plate V.

In any plane Triangle ABC, the Sides are proportional to the Sines of their oppoſite Angles, i. e. *S. C : AB : : S. A : BC, and S. C : AB: : S. B : AC; alſo S. B : AC : : S. A ; BC.* Fig. 10.

By Theo. 10. Sect. 1. the Half of each Side is the Sine of it's oppoſite Angle; but the Sines of thoſe Angles, in Tabular Parts, are proportional to the Sines of the ſame in any other Meaſure; and therefore the Sines of the Angles will be as the Halves of their oppoſite Sides: And ſince the Halves are as the Wholes, it follows, that the Sines of their Angles are as their oppoſite Sides, i. e. S. C : AB : : S. A : BC, *&c.* Q. E. D.

THEO. II.

In any plane Triangle ABC, the Sum of the two given Sides AB and BC, including a given Angle ABC, is to their Difference; as the Tangent of half the Sum of the two unknown Angles A and C is to the Tangent of half their Difference. Fig. 11.

Produce

Plate V.

Produce AB, and make HB = BC, and *join* HC: Let fall the Perpendicular BE, and that will biſect the Angle HBC (by Theo. 9. Sect. 1.) through B draw BD parallel to AC, and make HF = DC, and join BF; take BI = BA, and draw IG parallel to BD, or AC.

It is then plain, that AH will be the Sum, and HI the Difference of the Sides AB, and BC: And ſince HB = BC, and BE perpendicular to HC, therefore HE = EC (by Theo. 8. Sect. 1.); and ſince BA = BI, and BD and IG parallel to AC, therefore GD = DC = FH, and conſequently HG = FD, and ½HG = ½FD, or ED. Again, EBC, being half HBC, will be alſo half the Sum of the Angles A and C, by Theo. 4. Sect. 1. Alſo ſince HB, HF, and the included Angle H, are ſeverally equal to BC, CD, and the included Angle BCD; therefore (by Theo. 6. Sect. 1.) HBF = DBC = BCA (by Part 2. Theo. 3. Sect. 1.) and ſince HBD = A (by Part 3. Theo. 3. Sect. 1) and HBF = BCA; therefore FBD is the Difference, and EBD, half the Difference of the Angles A and C: Then making BE the Radius, it is plain that EC will be the Tan. of half the Sum, and ED the Tangent of half the Difference of the two unknown Angles A and C: Now IG being parallel to AC; AH : IH : : CH : GH. (by Cor. 1. Theo. 20. Sect. 1.) but the Wholes are as their Halves, i. e. AH : IH : : CE : ED, that is, as the Sum of the two Sides AB and BC, is to their Difference; ſo is the Tangent of half the Sum of the two unknown Angles A and C, to the Tangent of half their Difference. Q. E. D.

THEO.

Plate V.

THEO. III.

In any right-lined plane Triangle ABD; the Baſe AD, will be to the Sum of the other Side, AB, BD, as the Difference of thoſe Sides, is to the Difference of the Segments of the Baſe, made by the Perpendicular BE; viz. *the Difference between AE and ED. Fig.* 12.

Produce BD, till BG = AB the leſſer Leg; and on B as a Center, with the Diſtance BG or BA deſcribe a Circle AGHF; which will cut BD, and AD, in the Points H, and F: Then it is plain, that GD, will be the Sum, and HD, the Difference of the Sides AB and BD; alſo ſince AE = EF, (by Theo. 8. Sect. 1.) therefore FD, is the Difference of AE, and ED, the Segments of the Baſe: But (by Theo. 17. Sect. 1.) AD : GD : : HD : FD; that is the Baſe is to the Sum of the other Sides, as the Difference of thoſe Sides, is to the Difference of the Segments of the Baſe. Q. E. D.

THEO. IV.

If to half the Sum of two Quantities, be added half their Difference; the Sum will be the greateſt of them: and if from half the Sum be ſubtracted half their Difference, the Remainder will be the leaſt of them. Fig. 13.

Let the two Quantities be repreſented by AB, and BC; (making one continued Line;) whereof AB is the greateſt, and BC the leaſt; biſect the whole Line AC in E; and make AD = BC; then its

Plate V.

it is plain, that AC is the Sum, and BD, the Difference of the two Quantities; and AE, or EC, their half Sum, and DE or EB their half Difference. Now if to AE we add EB, we ſhall have AB the greateſt Quantity; and if from EC we take EB, we ſhall have BC, the leaſt Quantity. Q. E. D.

Cor. Hence, if from the greateſt of two Quantities, we take half the Difference of them, the Remainder will be half their Sum; or if to half their Difference be added the leaſt Quantity, their Sum will be half the Sum of the two Quantities.

OBLIQUE ANGULAR TRIGONOMETRY.

TWO *Sides, and an Angle oppoſite to one of them given, to find the other Angles and Side.*

In the Triangle ABC, there is given AB 240, *the Angle A* 46°. 30′, *and BC* 200; *to find the Angle C being acute, the Angle B, and the Side AC. Fig.* 14.

Geometrically.

Draw a blank Line, on which ſet AB 240 from a Scale of equal Parts; at the Point A, of the Line AB, make an Angle of 46°. 30′, by an infinite blank Line; with BC 200, from a like Scale of equal Parts that AB was taken, and one Foot in B deſcribe the Arc DC to cut the laſt blank Line in the Points D and C. Now if the Angle C had been required obtuſe; Lines from D to B, and

Plate V.

to A, would conſtitute the Triangle; but as it is required acute, draw the Lines from C to B, and to A; and the Triangle ABC is conſtructed. From a Line of Chords let the Angles B and C be meaſured; and AC from the ſame Scale of equal Parts, that AB and BC were taken; and you will have the Anſwer required.

By Calculation.

This is performed by Theo. 1. of this Sect, thus;

As BC	200	2.30103
is to the Sine of A	46° 30'	9.86056
So is AB	240	2.38021
		12.24077
To the Sine of C	60°. 31'	9.93974

180 — the Sum of the Angles A and C, will give the Angle B; by Cor. 1. Theo. 5. Sect. 1.

A 46°. 30'
C 60. 31

180 — 107. 10=72° 59'=B.

As the Sine of A	46°. 30'	9.86056
is to BC	200	2.30103
So is the Sine of B	72°. 59'	9.98056
		12.28159
To AC	263. 7	2.42103

By

Plate V.

By Gunter's Scale.

Extend from 200 to 240, on the Line of Numbers; that Diſtance will reach from 46°. 30ˡ on the Line of Sines, to 60°. 31ˡ for the Angle C.

Extend from 46°. 30ˡ, to 72°. 59ˡ, on the Line of Sines; that Diſtance will reach from 200 to 263.7, on the Line of Numbers for AC.

CASE 2.

Two Angles and a Side given, to find the other Sides.

In the Triangle ABC, there is the Angle A 46°. 30ˡ, *AB* 230, *and the Angle B* 37. 30 *given, to find AC and BC. Fig.* 15.

Geometrically.

Draw a blank Line, upon which ſet AB 230, from a Scale of equal Parts; at the Point A of the Line AB, make an Angle of 46°. 30ˡ, by a blank Line; and at the Point B of the Line AB, make an Angle of 37°. 30ˡ, by another blank Line; the Interſection of thoſe Lines gives the Point C, ſo is your Triangle ABC conſtructed. Meaſure AC and BC from the ſame Scale of equal Parts that AB was taken; and you have the Anſwer required.

By Calculation.

By (Cor. 1. Theo. 5. Section 1.) 180—the Sum of the Angles A and B = C.

Plate V.

A 46°. 30′
B 37. 30

180°———84. 00 = 96°. 00′ = C

By Def. 29. Sect. 1. The Sine of 96° = the Sine of 84°, which is the Supplement thereof; therefore inſtead of the Sine of 96°, look in the Tables for the Sine of 84°.

By Theo. 1. of this Sect.

As the Sine of C	96°. 00′	9.99761
is to AB	230	2.36173
So is the Sine of A	46°. 30′	9.86056
		12.22229
To BC	167.8	2.22468

As the Sine of C	96°. 00′	9.99761
is to AB,	230	2.36173
So is the Sine of B	37°. 30′	9.78445
		12.14618
to AC,	140 8	2.14857

By Gunter's *Scale.*

Extend from 84°, (which is the Supplement of 96°,) to 46°. 30′, on the Sines; that Diſtance will reach from 230, to 168, on the Line of Numbers for BC.

Extend

Plate V.

Extend from 84°, to 37°. 30′, on the Sines; that Extent will reach from 230, to 141, on the Line of Numbers for AC.

CASE 3.

Two Sides, and a contained Angle given; to find the other Angles and Side.

In the Triangle ABC, there is AB 240, *the Angle A* 36°. 40′, *and AC* 180 *given; to find the Angles C, and B, and the Side BC. Fig.* 16.

Geometrically.

Draw a blank Line, on which from a Scale of equal Parts, lay AB 240: at the Point A of the Line AB, make an Angle of 36°. 40′, by a blank Line; on which from A, lay AC 180, from the ſame Scale of equal Parts; Meaſure the Angles C, and B, and the Side BC, as before; and you have the Anſwer required.

By Calculation.

By Cor. 1. Theo. 5, Sect. 1. 180—the Angle A 36°. 40′, = 143°. 20′, the Sum of the Angles C, and B: Therefore half of 143°. 20′, will be half the Sum of the two required Angles, C, and B.

By Theo. 2, of this Sect.

As the Sum of the two Sides AB, and AC, 420
is to their Difference, 60

So

Plate V.

So is the Tangent of half the Sum of the two unknown Angles C and B	71°. 40′
to the Tangent of half their Difference	23°. 20′

By Theo. 4.

To half the Sum of the Angles C and B	71°. 40′
Add half their Difference, as now found	23. 20
The Sum is the greateſt Angle, or Ang. C	95. 00
Subtract, and you have the leaſt Angle, or B	48. 20

The Angles C, and B, being found; BC is had as before, by Theo. 1, of this Sect. Thus.

S. B : AC : : S : A : BC
48°. 30′ 180 36°. 40′ 143.9

By Gunter's *Scale.*

Becauſe the two firſt Terms are of the ſame Kind, extend from 420 to 60 on the Line of Numbers; lay that Extent from 45° on the Line of Tangents, and keeping the left Leg of your Compaſſes fixed, move the right Leg to 71°. 40′; that Diſtance laid from 45° on the ſame Line, will reach to 23°. 30′, the half Differenc of the required Angles. Whence the Angles are obtained as before.

The ſecond Proportion may be eaſily extended, for, from what has been already ſaid.

CASE

Plate V.

CASE 4.

The Sides given, to find the Angles.

In the Triangle ABC, there is given AB 64, *AC* 47, *BC* 34: *The Angles A, B, C, are required.* Fig. 17.

Geometrically.

The Conſtruction hereof muſt be manifeſt, from Prob. 1. Sect. 1.

By Calculation.

From the Point C, let fall the Perpendicular CD, on the Baſe AB; it will divide the Triangle into two right-angled ones, ADC, and CBD; as well as the Baſe AB, into the two Segments, AD and DB.

AC	47
BC	34
Sum	81
Difference	13

By Theo. 3, of this Sect.

As the Baſe, or longeſt Side, AB,	64
is to the Sum of the other Sides AC and BC,	81
So is the Difference of thoſe Sides,	13
to the Difference of the Segments of the Baſe AD, DB, — —	16.46

By

Plate V.

By Theo. 4, of this Sect.

To half the Baſe, or to half the Sum of the Segments AD and DB,	32
Add half their Difference, now found,	8.23
Their Sum will be the greateſt Segment AD,	40.23
Subtract, and their Difference will be the leaſt Segment DB, —	23.77

In the right-angled Triangle ADC; there is AC 47, and AD 40. 23, given, to find the Angle A.

This is reſolved by Caſe 4, of right-angled plane Trigonometry, thus

AD : R. : : AC : Sec. A.
40, 23 : 90° : : 47 : 31°. 08'

Or it may be had by finding the Angle ACD, the Complement of the Angle A; without a Secant, thus,

AC : R : : AD : S. ACD.
47 : 90° : : 40. 23 : 58°. 52'

90—58°. 52'=31°. 08', the Angle A.

Then, by Theo. 1, of this Sect.

BC : S. A : : AC : S. B.
34 : 31°. 08' : : 47 : 45°. 37'.

By Cor. 1. Theo. 5. Sect. 1. 180—the Sum of A and B, = C.

$$\begin{array}{ll} A & 31^\circ.\ 08' \\ B & 45.\ 37 \\ \hline \end{array}$$

$180 - 76.\ 45 = 103^\circ.\ 15'$, the Angle C.

By Gunter's *Scale.*

The firſt Proportion is extended on the Line of Numbers; and it is no Matter whether you extend from the firſt to the third, or to the ſecond Term, ſince they are all of the ſame Kind: If you extend to the ſecond, that Diſtance applied to the third, will give the fourth; but if you extend from the firſt to the third, that Extent will reach from the ſecond to the fourth.

The Methods of extending the other Proportions have been already fully treated of.

An Example in each Caſe of Oblique Angular Trigonometry.

$$1.\quad \text{Given, } \left\{ \begin{array}{lr} AC & 290 \\ C & 69^\circ.\ 30' \\ AB & 350 \end{array} \right\} \begin{array}{l} A \\ B \\ BC \end{array} \text{ required.}$$

$$2\quad \text{Given, } \left\{ \begin{array}{lr} C & 34^\circ.\ 20' \\ B & 128^\circ.\ 30' \\ AC & 3246 \end{array} \right\} \begin{array}{l} AB \\ \\ BC \end{array} \text{ required.}$$

$$3.\quad \text{Given, } \left\{ \begin{array}{lr} AC & 6 \\ C & 124^\circ.\ 30' \\ BC & 4.\ 5 \end{array} \right\} \begin{array}{l} A \\ B \\ AB \end{array} \text{ required.}$$

Plate V.

4. Given, $\left\{\begin{array}{ll} AB & 46 \\ AC & 92 \\ BC & 52 \end{array}\right\}\begin{array}{l} A \\ B \\ C \end{array}$ required.

Having thus gone through Plane Trigonometry, we ſhall now proceed to apply the ſame, in determining the Meaſures of inacceſſible Heights and Diſtances. And firſt,

---

Of

Of HEIGHTS.

Plate V.

THE Inſtrument of leaſt Expence for taking Heights, is a Quadrant, divided into 90 equal Parts or Degrees; and thoſe may be ſubdivided into Halves, Quarters, or Eighths, according to the Radius, or Size of the Inſtrument: It's Conſtruction will be evident by the Scheme thereof. (Fig. 18.)

From the Center of the Quadrant let a Plummet be ſuſpended by a Horſe Hair, or a fine Silk Thread; of ſuch a Length that it may vibrate freely, near the Edge of it's Arc: By looking along it's Edge AC, to the Top of the Object whoſe Height is required; and holding it perpendicular, ſo that the Plummet may neither ſwing from it, nor lie on it; the Degree then cut by the Hair, or Thread, will be the Angle of Altitude required.

If the Quadrant be fixed upon a Ball and Socket on a three-legged Staff, and if the Stem from the Ball be turned into the Notch of the Socket, ſo as to bring the Inſtrument into a perpendicular Poſition; the Angle of Altitude by this Means, can be acquired with much greater Certainty.

Plate V.

An Angle of Altitude may be alſo taken, by any of the Inſtruments uſed in Surveying; as ſhall be particularly ſhewn, when we treat of their Deſcriptions and Uſes.

Moſt Quadrants have a Pair of Sights fixed on the Edge AC, with ſmall circular Holes in them; which are uſeful in taking the Sun's Altitude, requiſite to be known in many aſtronomical Caſes; this is effected by letting the Sun's Ray, which paſſes thro' the upper Sight, fall upon the Hole in the lower one; and the Degree then cut by the Thread, will be the Angle of the Sun's Altitude; but thoſe Sights are uſeleſs for our preſent Purpoſe, for looking along the Quadrant's Edge to the Top of the Object will be ſufficient, as before.

PROB. I.

Plate 5. Fig. 19.

To find the Height of a perpendicular Object at one Station, which is on an horizontal Plane.

A Steeple.

Given, {
- The Angle of Altitude 53 Degrees.
- Diſtance from the Obſerver to the Foot of the Steeple, or the Baſe, 85 Feet.
- Height of the Inſtrument, or of the Obſerver, 5 Feet.

Required, the Height of the Steeple.

The Figure is conſtructed and wrought, in all Reſpects as Caſe 2 of right-angled Trigonometry; only there muſt be a Line drawn parallel to, and beneath

Plate V. Fig. 19.

beneath AB of 5 Feet for the Obſerver's Height, to repreſent the Plane upon which the Object ſtands; to which the Perpendicular muſt be continued, and that will be the Height of the Object.

Thus, AB is the Baſe, A the Angle of Altitude, BC the Height of the Steeple from the Inſtrument, or from the Obſerver's Eye, if he were at the Foot of it; DC the Height of the Steeple above the horizontal Surface.

Various Statings for BC, as in Caſe 2 of right-angled Plane Trigonometry.

90°
53 = A
37 = C

1. S. C : AB : : S. A : BC.
37° 85 53° 112. 8.

2. R. : AB : : T. A : BC.
90° 85 53° 112. 8.

3. T. C : AB : : R. : BC.
37° 85 90° 112. 8.

To BC, 112. 8
Add DB, 5. the Height of the Obſerver.

Their Sum is 117. 8 or 118 Feet, the Height of the Steeple required.

PROB.

PROB. II.

Plate V. Fig. 20.

To find the Height of a Perpendicular Object, on an horizontal Plane; by having the Length of the Shadow given.

Provide a Rod, or Staff, whoſe Length is given, let that be ſet perpendicular, by Help of a Quadrant, thus; apply the Side of the Quadrant AC, to the Rod, or Staff; and when the Thread cuts 90°, it is then perpendicular; the ſame may be done by a Carpenter's or Maſon's Plumb.

Having thus ſet the Rod or Staff perpendicular; meaſure the Length of it's Shadow, when the Sun ſhines, as well as the Length of the Shadow of the Object, whoſe Height is required; and you have the proper Requiſites given. Thus,

ab, the Length of the Shadow of the Staff, 15 Feet.

bc, the Length of the Staff, 10 Feet.

AB, the Length of the Shadow of the Steeple, or Object, 135 Feet.

Required BC, the Height of the Object.

The Triangles *abc*, ABC, are ſimilar, thus: The Angle $b = B$, being both right; the Lines *ac* AC, are parallel, being Rays, or a Ray of the Sun; whence the Angle $a = A$ (by Part 3. Theo. 3. Sect. 1.) and conſequently $c = C$. The Triangles being therefore mutually equiangular, are ſimilar, (by Theo. 16. Sect 1.) it will be,

ab : *bc* : : AB : BC.
15 10 135 90. the Steeple's Height required.

The foregoing Method is moſt to be depended on; however, this is mentioned for Variety ſake.

PROB. III.

To take the Altitude of a Perpendicular Object, at the Foot of a Hill, from the Hill's Side.

Turn the Center A of the Quadrant next your Eye, and look along the Side AC, or 90 Side, to the Top and Bottom of the Object; and noting down the Angles, meaſure the Diſtance from the Place of Obſervation to the Foot of the Object. Thus,

Given, {Angle to the Foot of the Object $55^{\circ}\frac{1}{4}$, or $55^{\circ}.\ 15^{l}$.
Angle to the Top of it $31^{\circ}\frac{1}{4}$ or $31^{\circ}.\ 15^{l}$.
Diſtance to the Foot of it 250 Feet.

Required the Height of the Object.

Geometrically.

Draw an infinite blank Line AD, at any Point in which A, make the Angle EAB of $55^{\circ}.\ 15^{l}$, and EAC of $31^{l}.\ 15^{l}$; lay 250 from A to B; from B draw the Perpendicular BE (by Prob. 7, of Geometry) croſſing AC in C; ſo will BC be the Height of the Object required.

In

Plate V. Fig. 21.

In the Triangle ABC there is given,

ABE the Complement of EAB to 90°, which is 34°. 45^{l}.

ACB the Difference of the given Angles 24°. 00^{l}.

The Side AB 250. Required BC.

This is performed as Caſe 2, of Oblique Angular Trigonometry. Thus,

180—the Sum of ABE 34°. 45^{l}, and CAB 24°. 00^{l} = ACB 121°. 15^{l}. Then,

S. ACB : AB : : S. CAB : BC.

121°. 15^{l} 250 24°. 00^{l} 119. The Height required.

PROB. IV.

To take the Altitude of a perpendicular Object, on the Top of a Hill, at one Station; when the Top and Bottom of it can be ſeen from the Foot of the Hill.

Plate V. Fig. 22.

As in Prob. 1; take an Angle to the Top, and another to the Bottom of the Object; and meaſure from the Place of Obſervation to the Foot of the Object, and you have all the given Requiſites. Thus.

A Tower on A Hill.

Given, { Angle to the Bottom 48°. 30^{l}.
Angle to the Top 67°. 00^{l}.
Diſt. to the Foot of the Object 136 Feet.
Required, the Height of the Object.

Geo-

Plate V. Fig. 22.

Geometrically.

Make the Angle DAB = 48°. 30′, and lay 136 Feet, from A to B; from B let fall the Perpendicular BD, and that will be the Height of the Hill: Produce BD upwards by a blank Line: Again, at A make the Angle DAC = 67°. 00′ by a blank Line, and from C where that croſſes the Perpendicular produced, draw the Line CB, and that will be the Height of the Object required.

Let AC be drawn.
In the Triangle ABC, there is given,
The Angle ACD the Complement of DAC = 23° 00′.
CAB the Difference between the two given Angles = 18°. 30′.
And the Side AB 136. To find BC.

S.C : AB :: S.CAB : BC.
23° 136 18°. 30′ 110½

If BD were wanted, it is eaſily obtained, by the firſt Caſe of Right-angled Plane Trigonometry.

PROB. V.

To take an inacceſſible perpendicular Altitude, on an horizontal Plane.

Plate V. Fig. 23.

This is done at two Stations, thus:

Plate V. Fig. 23.

Let DC be a Tower which cannot be approached by means of a Mote or Ditch nearer than B; at B, take an Angle of Altitude to C: Meaſure any convenient Diſtance backward to A, which note down: At A take another Angle to C; ſo have you the given Requiſites, thus;

Given, { Firſt Angle 55°. 00'
Stationary Diſtance 87 Feet;
Second Angle 37°. 00'.

The Height of the Tower CD, is required.

Geometrically.

Upon an infinite blank Line, lay off the ſtationary Diſtance 87, from A to B; from B, ſet off your firſt; and from A, your ſecond Angle: From C, the Point of Interſection of the Lines which form theſe Angles, let fall the Perpendicular CD; and that will be the Height of the Object required.

The external Angle CBD, of the Triangle ABC; is equal to the two internal oppoſite ones, A, and ACB (by Theo. 4. Sect. 1.): Wherefore if one of the internal oppoſite Angles, be taken from the external Angle; the Remainder will be the other internal oppoſite one, thus;

$$\text{CBD } 55^\circ - \text{A } 37^\circ = \text{ACB } 18^\circ.$$

Therefore in the Triangle ABC; we have the Angles A, and ACB, with the Side AB given; to find BC.

S. ACB

Plate V. Fig. 23.

S. ACB : AB : : S. A : BC.
18° 87 37° 169.4

Having found BC, we have in the Triangle BCD; the Angle CBD 55°, conſequently BCD 35°, and BC 169.4; to find DC.

This is performed by Caſe the firſt, of Right-angled Trigonometry, three ſeveral Ways; thus,

1. R : BC : : S. CBD : DC.
90° 169.4 55° 138.8. The Height required.

2. Sec. CBD : BC : : T. CBD : DC.
55° 169.4 55° 138.8. The Height required.

3. Sec. BCD : BC : : R : CD.
35° 169.4 90° 138.8. The Height required.

If BD, the Breadth of the Mote, were required; it may alſo be found, by three different Statings, as in the firſt Caſe of Right-angled Plane Trigonometry.

PROB. VI.

Plate V. Fig. 24.

Let BC a May-pole, whoſe Height is 100 Feet, be broken at D; the upper Part of which DC, falls upon an horizontal Plane, ſo that it's extremity C, is 34 Feet from the Bottom or Foot of the Pole.

Required the Segments BD and DC.

Geometrically.

Lay 34 Feet from A to B; on B erect the Perpendicular BC of 100 Feet, and draw AC: Biſect AC, (by Prob. 4. Sect. 1.) with the Perpendicular Line EF; and from D, where it cuts the Perpendicular BC, draw AD, which will be the upper Segment; and DB will be the lower.

By Cor. to Lemma, preceding Theo. 7. Sect. 1. AD = DC; and (by the Lemma) the Angle C = CAD.

In the Triangle ABC, find C, as in Caſe 6, of Right-angled Trigonometry, thus;

1. BC : R : : AB : T. C = GAD.
   100 90° 34 $18^{\circ}.\ 47^{\prime}$.

By Theo. 4. Sect. 1. The external Angle ADB = $37^{\circ}.\ 34^{\prime}$, or to twice the Angle C; i. e. to C, and GAD.

Then in the Triangle ABD, there is ADB $37^{\circ}.\ 34^{\prime}$, therefore alſo it's Complement DAB $52^{\circ}.\ 26^{\prime}$, and AB 34 given, to find AD, and AB.

By the ſecond Caſe of Rect-angular Trigonometry.

2. S. ADB : AB : : R : AD, or DC.
   $37 \cdot 34^{\prime}$ 34 90° 55.77

   BC—DC = BD.
   100—55.77 = 44.23 required.

Theſe may be had from other Statings, as in the ſecond Caſe aforeſaid.

PROB. VII.

To take the Altitude of a perpendicular Object on a Hill, from a Plane beneath it.

Plate V. Fig. 25.

This is done at two Stations, thus;

Let the Height DC, of a Wind-mill on a Hill be required.

From any Part of the Plane whence the Foot of the Object can be ſeen, let Angles be taken to the Foot, and Top; meaſure thence any convenient Diſtance towards the Object, and at the End thereof, take another Angle to the Top; And you have the proper Requiſites, thus;

Firſt Station.	Angle to the Foot	DAB	21°. 00ʹ.
	Angle to the Top	CAB	35°. 00ʹ.
	Stationary Diſtance	AB	104 Feet,
Second Station.	Angle to the Top		48°. 30ʹ.

DC, and DE required.

Geometrically.

On an infinite blank Line, lay the ſtationary Diſtance AB 104 Feet; from A, ſet off the ſecond, and from B, the third given Angle; and from the

inter-

Plate V. Fig. 25.

interſecting Point C of the Lines formed by them; let fall the Perpendicular CE: From A ſet off the firſt Angle, and the Line formed by it will determine the Point D. Thus have we the Height of the Hill, as well as that of the Windmill.

The Angle CBE—A = ACB, as before.

In the Triangle ABC, find BC thus,

1. S. ACB : AB : : S. A : BC.
13°. 30′ 104 35°. 00′ 255. 5.

In the Triangle CBE, find CE thus,

2. R. : BC : : S. CBE : CE.
90° 255.5 48°. 30′ 191.4.

In the ſame Triangle find BE thus,

3. R. : BC : : S. BCE : (or Comp. of CBE) BE.
90° 255.5 41°. 30′ 169.3.

AB+BE = AE, or 104+169.3 = 273.3 = AE.

In the Triangle AED, find DE thus,

4. S. ADE : (or Comp. of DAB) AE : : S. DAE
69° 273.3 21
: DE.
104.9, required.

CE—DE = CD, or 191.4 — 104.9 = 86.5 = CD, required.

CE,

Plate V. Fig. 25.

CE, BE, or DE, may be found by other various Statings, as ſet forth in the firſt and ſecond Caſes of Rectangular Trigonometry.

PROB. VIII.

To find the Length of an Object, that ſtands obliquely on the Top of a Hill, from a Plane beneath.

Let CD be a Tree whoſe Length is required.

This is done at two Stations.

Make a Station at B, from whence take an Angle to the Foot, and another to the Top of the Tree; meaſure any convenient Diſtance backward to A, from whence alſo let an Angle be taken to the Foot, and another to the Top; and you have the Requiſites given. Thus,

Firſt Station. Angle to the Foot EBD = $36^\circ.\ 30'$
Angle to the Top EBC = $44^\circ.\ 30'$.
Stationary Diſtance AB 104 = Feet.

Second Station. Angle to the Foot EAD = $24^\circ.30'$.
Angle to the Top EAC = $32^\circ.\ 00'$.

Let DC and DE be required.

The Geometrical Conſtructions of this and the next Problem are omitted; as what has been already ſaid, and the Figures are looked upon as ſufficient Helps.

EBC—A = ACB, or $44^\circ.\ 30' - 32^\circ = 12^\circ.\ 30'$, as before.

Plate V. Fig. 26.

In the Triangle ABC, find BC. Thus:

1. S. ACB : AB : : S. A : BC.
$12^{\circ}.\ 30^{l}$ 104 32° 254.7.

EBD—EAD = ADB, or $36^{\circ}.\ 30^{l}$—$24^{\circ}.\ 30^{l}$ = $12^{\circ}.00^{l}$.

In the Triangle ADB, find BD. Thus:

2. S. ADB : AB : : S. DAB : DB.
$12^{\circ}.\ 00^{l}$ 104 $24^{\circ}.\ 30^{l}$ 207.4.

CBE—DBE = CBD, or $44^{\circ}.\ 30^{l}$—$36^{\circ}.\ 30^{l}$ = $8^{\circ}.\ 00^{l}$.

In the Triangle CBD there is given, CB 254.7, DB 207.4, and the Angle CBD $8^{\circ}.\ 00^{l}$; to find DC.

This is performed as Caſe 3, of Oblique-angular Trigonometry, thus:

3. BC+BD : BC—BD : : T. of $\frac{1}{2}$ BDC+BCD :
462.1 47.3 $86^{\circ}.\ 00^{l}$
T. of $\frac{1}{2}$ BDC—BCD.
$55^{\circ}.\ 40$.

$86^{\circ}.\ 00^{l}$+$55^{\circ}.\ 40^{l}$ = $141^{\circ}.\ 40^{l}$ = BDC.
$86^{\circ}.\ 00^{l}$—$55^{\circ}.\ 40^{l}$ = $30^{\circ}.\ 20^{l}$ = BCD.

4. S. BCD : BD : : S. CBD : DC.
$30^{\circ}.\ 20^{l}$ 207.4 $8^{\circ}.\ 00^{l}$ 57.15 Length of the Tree.

To find DE, in the Triangle DBE.

Say R. : BD : : S. DBE : DE.
90° 207.4 $36^{\circ}.\ 30^{l}$ 123.4. Height of the Hill.

PROB.

PROB. IX.

To find the Height of an inacceſſible Object CD, on a Hill BC, from Ground that is not horizontal.

Plate VI. Fig. 1.

From any two Points, as G and A, whoſe Diſtance GA is meaſured, and therefore given; let the Angles HGD, BAD, BAC, and EAG, be taken: Becauſe GH is parallel to EA (by Part 2. Theo. 3. Sect. 1.) the Angle HGA=EAG; therefore EAG +HGD=AGD: And (by Cor. 1. Theo. 1. Sect. 1.) 180—the Sum of EAG and BAD=GAD: And (by Cor. 1. Theo. 5. Sect. 1.) 180—the Sum of the Angles AGD and GAD=GDA: Thus we have the Angles of the Triangle AGD, and the Side AG given; thence (by Caſe 2, of Ob. Trig.) AD may be eaſily found. The Angle DAB—CAB =DAC, and 90°—BAD=ADC; and 180°—the Sum of DAC and ADC=ACD: So have we the ſeveral Angles of the Triangle ACD given, and the Side AD; whence (by Caſe 2, Obl. Trig.) CD may be eaſily found. We may alſo find AC, which with the Angle BAC, will give CB, the Height of the Hill.

The Solutions of the ſeveral Problems in Heights and Diſtances, by *Gunter*'s *Scale*, are omitted; becauſe every particular Stating has been already ſhewn by it, in the Rectangular and Oblique-angular Trigonometry.

Of DISTANCES.

ANY of the Inſtruments uſed in Surveying, will give you the Angles or Bearings of Lines; which will be particularly ſhewn, when we come to treat of them.

PROB. I.

Plate VI. Fig. 2.

Let A and B be two Houſes on one Side of a River, whoſe Diſtance aſunder is 293 Perches: There is a Tower at C, on the other Side of the River, that makes an Angle at A, with the Line AB of 53°. 20'; and another at B, with the Line BA of 66°. 20': Required the Diſtance of the Tower from each Houſe, *viz.* AC and BC.

This is performed as Caſe 2, of Oblique-angled Trigonometry, thus:

1. S. C : AB : : S. A : BC.
60°. 20' 293 53°. 20' 270.5.

2. S. C : AB : : S. B : AC.
60.° 20' 293 66°. 20' 308.8.

PROB.

PROB. II.

Plate VI. Fig. 11.

Let B, and C, be two Houſes whoſe direct Diſtance aſunder BC, is inacceſſible: However it is known, that a Houſe at A, is 252 Perches from B, and 230 from C; and that the Angle BAC, is found to be 70°. What is the Diſtance BC, between the two Houſes?

This is performed as Caſe 3, of Oblique-angular Trigonometry, thus:

1. AB+AC : AB—AC : T. of ½ C+B :
   482 22 $55^{\circ}.00^{\prime}$

   T. of ½ C—B.
   $3^{\circ}.44^{\prime}$.

$55^{\circ}+3^{\circ}.44^{\prime}=58^{\circ}.44^{\prime}.=C$. $55^{\circ}-3^{\circ}.44^{\prime}=51^{\circ}.16^{\prime}$
$=B$

2. S. C : AB : : S. A : BC.
   $58^{\circ}.44^{\prime}$ 252 70° 279.9

PROB. III.

Plate VI. Fig. 3.

Suppoſe ABC a triangular Piece of Ground, which by an old Survey we find to be thus: AB 260, AC 160, BC 150, Perches: The mearing Lines AC, and BC, are deſtroyed or plowed down, and the Line AB, only remaining. What Angles muſt be ſet off at A, and B, to run new Mearings by, exactly where the old ones were?

This is performed as in Caſe 4, of Oblique-angled Trigonometry. Thus,

1. AB : AC+BC : : AC—BC : AD—DB.
260 310 10 11.92.

130 + 5.96 = 135.96 = AD.
130—59.6 = 124.04 = DB.

2. AD : R : : AC : Sec. A.
136 90° : : 160 31°. 47^{l}.

3 BC : S. A : : AC : S. B.
150 31°. 47^{l} 160 34°. 40^{l}.

PROB. IV.

Plate VI. Fig. 4.

Let D, and C, be two Trees in a Bog, to which you can have no nearer Acceſs than at A, and B: there is given, DAB 100°, CAB 36°. 30^{l}, CBA 121°, DBA 49°, and the Line AB 113 Perches. Required the Diſtances of the Trees DC.

180°—the Sum of DBA and DAB = ADB = 31°.
180°—the Sum of CAB and CBA = ACB = 22. 30.

In the Triangle ABD, find DB, thus:

1. S.ADB : AB : SDAB : DB.
31° 113 : : 100° 216.

And the Triangle ABC, find BC thus,

2. S. ACB : AB : : S. CAB : BC.
$22^{\circ}.\ 30^{1}$ 113 $36^{\circ}.\ 30^{1}$ 175.6.

In the Triangle DBC, you have DBC = ABC—ABD = 72°; likewiſe the Sides BD, BC, as before found given, to find DC.

3. BD+BC : BD—BC : : T. of ½ DCB+CDB :
391.6 40.4 54°
T. of ½ DCB—CDB.
$8^{\circ}.\ 05^{1}$.

$54^{\circ} + 8^{\circ}.\ 05^{1} = 62^{\circ}.\ 05^{1} =$ DCB.
$54^{\circ} - 8^{\circ}.\ 05^{1} = 45^{\circ}.\ 55^{1} =$ CDB.

4. S. CDB : BC : : S. DBC : DC.
$45^{\circ}.\ 55^{1}$ 176.3 72° 215.8.

LEMMA.

Plate VI. Fig. 10.

If from a Point C, of a Triangle ABC, inſcribed in a Circle, there be a Perpendicular CD let fall upon the oppoſite Side AB: That Perpendicular is to one of the Sides including the Angle, as the other Side, including the Angle, is to the Diameter of the Circle, i. e. DC : AC : : CB : CE.

Let the Diameter CE be drawn and join EB; it is plain the Angle CEB = CAB, (by Cor. 2. Theo. 7. Sect. 1.) and CBE is a right Angle (by Cor. 5. Theo. 7. Sect. 1.) and = ADC: Whence ECB =

ACD. The Triangles CEB, CAD, are therefore mutually equiangular, and (by Theo. 16. Sect. 1.) DC : AC : : CB : CE, or DC : CB : : AC : CE. Q. E. D.

PROB. V.

Plate VI. Fig. 5.

Let three Gentlemens Seats, A, B, C, be ſituate in a triangular Form: There is given AB 2.5 Miles, AC 2.3, and BC 2. It is required to build a Church at E, that ſhall be equi-diſtant from the Seats A, B, C: What Diſtance muſt it be from each Seat, and by what Angle may the Place of it be found.

Geometrically.

By Prob. 15. Sect. 1. Find the Center of a Circle that will paſs through the Points A, B, C; and that will be the Place of the Church; the Meaſure of which, to any of theſe Points, is the Anſwer for the Diſtance: Draw a Line from any of the three Points to the Center, and the Angle it makes with either of the Sides that contain the Angle it was drawn to; that Angle layed off by the Direction of an Inſtrument, on the Ground, and the Diſtance before found being ranged thereon, will give the Place of the Church required.

By Calculation.

1. AB : AC+BC : : AC—BC : AD—DB.
2.5 4.3 .3 .516.

1.25+.258 = 1.508 = AD.

By Cor. 2. Theo. 14. Sect. 1. The Square Root of the Difference of the Squares of the Hypothenuſe AC, and given Leg AD, will give DC.

i. e. $5.29 - 2.274064 = 3.015936.$

It's Square Root is $1.736 =$ CD.

Then by the preceding Lemma,

2. CD : AC : : CB : the Diameter.
1.736 2.3 2 2.65.

the half of which, *viz.* 1.325 is the Semi-diameter, or Diſtance of the Church from each Seat, that is AE, CE, BE.

From the Center E, let fall a Perpendicular upon any of the Sides as EF, and it will biſect in E: (by Theo. 8. Sect. 1.)

Wherefore AF $=$ CF $= \frac{1}{2}$AC $= 1.15$.

In the right-angled Triangle AFE, you have AF 1.15, and AE the Radius 1.325 given, to find FAE, thus,

3. AF : R. : : AE : Sec. FAE.
1.15 90° 1.325 $29^\circ.\ 47^{l}$.

Wherefore directing an Inſtrument to make an Angle of $29^\circ.\ 47^{l}$, with the Line AC; and meaſuring 1.325 on that Line of Direction, will give the Place of the Church, or the Center of a Circle that will paſs through A, B, and C.

The above Angle FAE, may be had without a Secant as before, thus,

AE : R. : : AF : S. AEF.
1.325 90° 1.15 60°. 13'.

It's Complement 29°. 47', will give FAE, as before.

The Queſtions that may be propoſed on this Head, being innumerable, we have choſen to give only a few of the moſt uſeful.

Of

SECT. III.

Containing a particular Deſcription of the ſeveral Inſtruments uſed in Surveying, with their reſpective Uſes. And firſt,

Of the CHAIN.

THE Stationary Diſtance, or Mearings of Ground, are meaſured either by Mr. *Gunter*'s Chain of four Poles or Perches, which conſiſts of 100 Links; (and this is the moſt natural Diviſion) or by one of 50 Links, which contains Two Poles or Perches: But becauſe the Length of a Perch differs in many Places, therefore the Lengths of Chains and their reſpective Links will differ alſo.

The *Engliſh Statute-Perch* is 5 ½ Yards, the Two-Pole-Chain is 11 Yards, and the Four-Pole one is 22 Yards: Hence the Length of a Link in a *Statute Chain* is 7.92 Inches.

There are other Perches uſed in different Parts of *England*, as the Perch of *Wood land Meaſure*, which is 6 Yards; that of *Church-land Meaſure*, which is 7 Yards (or the ſame with the *Plantation Perch*) and the *Foreſt Meaſure Perch*, which is 8 Yards.

The *Iriſh*, or *Plantation Perch* is 7 Yards, as before; the Two-Pole Chain is 14; and the Four-Pole one is 28 Yards: Hence the Length of a *Plantation Chain* is 10.08 Inches.

The *Scotch Perch* is 18 ½ Feet, or 6 ⅙ Yards, or 6 *Scot*'s Ells. In the Shire of *Cunningham* in *Scotland*, their Perch is 18 ¾ Feet, and this Perch is uſed in ſome few Places in the North Part of this Kingdom, as the Statute Perch is in ſome other Parts.

For the more ready reckoning the Links of a Four-Pole Chain, there is a large Ring, or ſometimes a round Piece of Braſs fixed at every 10 Links; and at 50 Links, or in the Middle, there are 2 large Rings. In ſuch Chains as have a Braſs Piece at every 10 Links, there is the Figure 1 on the firſt Piece, 2 on the ſecond, 3 on the third, *&c.* to 9. By leading therefore that End of the Chain forward, which has the leaſt Number next it, he who carries the hinder End may eaſily determine any Number of Links: Thus, if he has the Braſs Piece Number 8, next to him, and 6 Links more in a Diſtance, that Diſtance is 86 Links. After the ſame Manner 10 may be counted for every large Ring of a Chain, which has not Braſs Pieces on it; and the Number of Links is thus readily determined.

The Two-Pole Chain has a large Ring at every 10 Links, and in its Middle, or at 25 Links, there are 2 large Rings; ſo that any Number of Links may be the more readily counted off, as before.

The Surveyor ſhould be careful to have his Chain meaſured before he proceeds on Buſineſs, for the

Rings are apt to be open by frequent uſing it, and it's Length is thereby encreaſed, ſo that no one can be too circumſpect in this Point.

In meaſuring a Stationary Diſtance, there is an Object fix'd in the extream Point of the Line to be meaſured; this is a Direction for the hinder Chainman to govern the foremoſt one by, in order that the Diſtance may be meaſured in a right Line; for if the hinder Chainman cauſes the other to cover the Object, it is plain the foremoſt is then in a right Line towards it. For this Reaſon it is neceſſary to have a Perſon that can be relied on, at the hinder End of the Chain, in order to keep the foremoſt Man in a right Line; and a Surveyor who has no ſuch Perſon ſhould chain himſelf. The Inaccuracies of moſt Surveys ariſe from bad Chaining, that is, from ſtraying out of the right Line, as well as from other Omiſſions of the hinder Chainman: No Perſon therefore ſhould be admitted at the hinder End of the Chain, of whoſe Abilities in this reſpect, the Surveyor was not previouſly ſatisfied and convinced; ſince the Succeſs of the Survey, in a great Meaſure depends on his Care and Skill.

In ſetting out to meaſure any ſtationary Diſtance, the fore Man of the Chain carries with him 10 Iron Pegs pointed, each about ten Inches long; and when he has ſtretched the Chain to it's full Length, he at the Extremity thereof ſticks one of thoſe Pegs perpendicularly in the Ground; and leaving it there, he draws on the Chain 'till the hinder Man checks him when he arrives at that Peg: The Chain being again ſtretched, the fore Man ſticks down another Peg, and the hind Man takes up the former; and thus they proceed at every Chain's Length contained in the Line to be meaſured, counting the ſurplus Links contained between the laſt Peg, and the Object at the Termination of the Line, as before: So that the Number of Pegs taken up by the hinder Chainman, expreſſes the Number of Chains; to which if the odd Links be annexed, the Diſtance Line required in Chains and Links is obtained, which muſt be regiſtred in the Field-Book, as will hereafter be ſhewn.

If the Diſtance exceeds 10, 20, 30, *&c.* Chains, when the Leader's Pegs are all exhauſted, the hinder Chainman, at the Extremity of the 10 Chains, delivers him all the Pegs; from whence they proceed to meaſure as before, 'till the Leader's Pegs are again exhauſted, and the hinder Chainman at the Extremity of theſe 10 Chains again delivers him the Pegs; from whence they proceed to meaſure the whole Diſtance Line in the like Manner: then 'tis plain, that the Number of Pegs the hinder Chainman has, being added to 10, if he had delivered all the Pegs once to the Leader, or to 20 if twice, or to 30 if thrice, *&c.* will give the Number of Chains in that Diſtance; to which if the ſurplus Links be added, the Length of the ſtationary Diſtance is known in Chains and Links.

It is cuſtomary, and indeed neceſſary, to have Red, or other coloured Cloth fixed to the Top of each Peg, that the hinder Man at the Chain may the more readily find them; otherwiſe in chaining thro' Corn, high Graſs, Briars, Ruſhes, Potatoes, *&c.* it would be extremely difficult to find the Pegs which the Leader puts down: By this Means no Time is loſt, which otherwiſe muſt be if no Cloths are fixed to the Pegs as before.

It will be neceſſary here to obſerve, that all ſlant, or inclined Surfaces, as Sides of Hills, are meaſured horizontally, and not on the Plane or Surface of the Hill, and is thus effected; Let

Plate VIII. Fig. 4.

Let ABC be a Hill, the hindmoſt Chainman is to hold the End of the Chain perpendicularly over the Point A, (which he can the better effect with a Plummet and Line, than by letting a Stone drop, which is moſt uſual) as *d* is over A, while the Leader puts down his Peg at *e*: The Eye can direct the horizontal Poſition near enough, (but if greater Accuracy were required, a Quadrant applied to the Chain, would ſettle that). In the ſame Manner the reſt may be chained up and down; but in going down it is plain the Leader of the Chain muſt hold up the End thereof, and the Plummet thence ſuſpended will mark the Point where he is to ſtick his Peg. The Figure is ſufficient to render the whole evident; and to ſhew that the Sum of the Chains will be the horizontal Meaſure of the Baſe of the Hill; for $de = Ao$, $fg = op$, $bi = pq$, &c. therefore $de + fg + bi$, &c. $= Ao + op + pq$, &c. $=$ AC, the Baſe of the Hill. If an whole Chain cannot be carried horizontally, half a one or leſs may, and the Sum of theſe half Chains, or Links, will give the Baſe as before.

If the inclined Side of a Hill be a plane Surface, the Angle of the Hill's Inclination may be taken, and the ſlant Height may be meaſured on the Surface; and thence (by Caſe 1, of right-angled Trigonometry,) the horizontal Line anſwering to the Top may be found; and if we have the Angle of Inclination given on the other Side, with thoſe already given; we can find the horizontal Diſtance acroſs the Hill, by Caſe 2, of Oblique Trigonometry.

All inclined Surfaces are conſidered as horizontal ones; for all Trees which grow upon any inclined

Surface, do not grow perpendicular thereto, but to the Plane of the Horizon: Thus if A*d*, *ef*, *gb*, &c. were Trees on the Side of a Hill, they grow perpendicular to the horizontal Baſe AC, and not to the Surface AB: Hence the Baſe will be capable to contain as many Trees as are on the Surface of the Hill, which is manifeſt from the Continuation of them thereto. And this is the Reaſon that the Area of the Baſe of a Hill is conſidered to be equal in Value to the Hill itſelf.

Beſides, the Irregularities of the Surfaces of Hills in general are ſuch, that they would be found impoſſible to be determined by the moſt able Mathematician. Certain regular curve Surfaces have been inveſtigated with no ſmall Pains by the moſt Eminent: Therefore an Attempt to determine in general the Infinity of irregular Surfaces which offer themſelves to our View, to any Degree of Certainty, would be idle and ridiculous, and for this Reaſon alſo the horizontal Area is only attempted.

Again, if the circumjacent Lands of a Hill be planned or map'd, it is Evident we will have a Plan of the Hill's Baſe in the Middle: But were it poſſible to put the Hill's Surface in Lieu thereof, it would extend itſelf into the circumjacent Lands; and render the Whole an Heap of Confuſion: So that if the Surfaces of Hills could be determined, no more than the Baſe could be map'd.

Roads are uſually meaſured by a Wheel for that Purpoſe, to which there is fixed a Machine, at the End whereof there is a Spring, which is ſtruck by a Peg in the Wheel once in every Rotation; by this Means the Number of Rotations is known.

If ſuch a Wheel were 3 Feet 4 Inches in Diameter, one Rotation would be 10½ Feet, which is half a Plantation Perch; and becauſe 320 Perches make a Mile, therefore 640 Rotations will be a Mile alſo: and the Machinery is ſo contrived, that by Means of a Hand, which is carried round by the Work, it points out the Miles, Quarters, and Perches, or ſometimes the Miles, Furlongs, and Perches.

Or Roads may be meaſured by a Chain more accurately; for 80 Four-Pole, 160 Two-Pole Chains, or 320 Perches make a Mile as before: And if Roads are meaſured by a Statute Chain, it will give you the Miles *Engliſh*, but if by a Plantation Chain, the Miles will be *Iriſh*. Hence an *Engliſh* Mile contains 1760, and an *Iriſh* Mile 2240 Yards; and becauſe 14 half Yards is an *Iriſh*, and 11 half Yards is an *Engliſh* Perch, therefore 11 *Iriſh* Perches, or *Iriſh* Miles, are equal to 14 *Engliſh* ones.

Since ſome Surveys are taken by a Four-Pole, and others by a Two-Pole Chain; and as Ground for Houſes is meaſured by Feet, we will ſhew how to reduce one to the other, in the following Problems.

PROB. I.

To reduce Two-Pole Chains and Links to Four-Pole ones.

If the Number of Chains be even, the half of them will be Four-Pole ones, to which annex the Links given, thus,

	Ch.	L.	
1. In	16.	37	of Two-Pole Chains, how many Four-Pole ones?

	Ch.	L.
Anſwer,	8.	37.

But if the Number of Chains be odd, take the half of them for Chains, and add 50 to the Links, and they will be Four-Pole Chains and Links, thus,

	Ch.	L.	
2. In	17.	42	of Two-Pole Chains, how many Four-Pole ones?

	Ch.	L.
Anſwer,	8.	92.

PROB. II.

To reduce Four-Pole Chains and Links, to Two-Pole ones.

Double the Chains, to which annex the Links, if they be leſs than 50; but if they exceed 50, double the Chains, add 1 to them, and take 50 from the Links, and the Remainder will be the Links, thus,

	Ch.	L.	
1. In	8.	37	of Four-Pole Chains, how many
	2		Two-Pole ones?
	16.	37	Anſwer.

		Ch. L.	
2.	In	8. 82	of Four-Pole Chains, how many
		2. 50	Two Pole ones?
		17. 32	Anſwer.

PROB. III.

To reduce Four-Pole Chains and Links, to Perches and Decimals of a Perch.

The Links of a Four-Pole Chain are decimal Parts of it, each Link being the hundredth Part of a Chain; therefore if the Chains and Links be multiplied by 4, (for 4 Perch is a Chain) the Product will be the Perches and decimal Parts of a Perch. Thus,

	Ch.	L.	
How many Perches in	13.	64	of Four-Pole Chains?
		4	
Anſwer	54.	56	Perches.

PROB. IV.

To reduce Two-Pole Chains and Links, to Perches and Decimals of a Perch.

They may be reduced to Four-Pole ones, (by Prob. 1.) and thence to Perches and Decimals, (by the laſt.) or, If the Links be multiplied by 4, carrying one to the Chains, when the Links are, or exceed 25; and the Chains by 2, adding one if Occaſion be: The Product will be Perches, and Decimals of a Perch. Thus,

C. L.
1. In 17. 21 of Two-Pole Chains how many
2. 4 Perches?

Anſwer 34. 84 Perches?

C. L.
2. In 15. 38 of Two-Pole Chains, how many
2. 4 Perches?

Anſwer 31. 52 Perches.

PROB. V.

To reduce Perches, and Decimals of a Perch, to Four-Pole Chains and Links.

Divide by 4, and cut two Decimals from the Quotient, and that will be Four-Pole Chains and Links. Thus,

In 31.52 Perches, how many Four-Pole Chains and Links?

C. L.
4)31.52(7. 88 Anſwer.

35

32

..

PROB. VI.

To reduce Perches and Decimals of a Perch, to Two-Pole Chains and Links.

The Perches may be reduced to Four-Pole Chains, (by the laſt) and from thence to Two-Pole Chains, (by Prob. 2.) or,

Divide the whole Number by 2, the Quotient will be Chains; to the Remainder annex the given Decimals, and divide by 4, the laſt Quotient will be the Links. Thus,

In 31.52 Perches, how many Two-Pole Chains and Links?

```
            C.  L.
2)31.52(15. 38  Anſwer.
  ------
    11
  ------
  4) 152 (38
  ------
     32
     ---
     ..
```

PROB. VII.

To reduce Chains and Links, to Feet and Decimal Parts of a Foot.

If they are Two-Pole Chains, reduce them to Four-Pole ones: (by Prob. 1.) Theſe being multiplied by the Feet in a Four-Pole Chain; will give the Feet, and Decimals of a Foot. Thus,

C. L.
In 17. 21 of Two-Pole Plantation-Chains, how many Feet?

C. L.
is 8. 71 of Four-Pole Chains.
84

3484
6968

Feet 731.64
12

Inches 7.68
4

2.72

Feet Inches
Anſwer 731. 7½

PROB. VIII.

To reduce Feet and Inches to Chains and Links.

Reduce the Inches to the Decimal of a Foot, and annex that to the Feet; that divided by the Feet in a Four-Pole Chain, will give Four-Pole Chains and Links in the Quotient: Theſe may be reduced to Two-Pole Chains and Links, if required, by Prob. 2. Thus,

Feet Inches
In 217. 9 how many Two-Pole Plantation-Chains?

12)9.00(.75 the Decimal of 9 Inches.
84

60
---

84)

C. L.
84)217.75(2. 59 of Four-Pole Chains.
497
C. L.
775 or 5. 09 of two Pole Chains.
19

How to take a Survey by the CHAIN *only.*

PROB. I.

To ſurvey a Piece of Ground, by going round it, and the Method of taking the Angles of the Field, by the Chain only.

Plate VI. Fig. 5.

Let A B C D E F G be a Piece of Ground to be ſurveyed: beginning at the Point A, let one Chain be laid in a direct Line from A towards G, where let a Peg be left as at *c*, and again the like Diſtance from A in a direct Line towards B, where another Peg is alſo to be left as at *d*: Let the Diſtance from *d* to *c* be meaſured, and placed in the Field-Book, in the ſecond Column under the Denomination of Angles, in a Line with Station No. 1; and in the ſame Line under the Title of Diſtances, in the third Column, let the meaſure of the Line AB in Chains and Links be inſerted. Being now arrived at B, let one Chain be laid in a direct Line from B towards A, where let a Peg be left, at *f*, and again, the like Diſtance from B in a direct Line towards C, where let alſo another Peg be left, as at *e*; the Diſtance from *e* to *f* is to be inſerted in the Field-Book, in the ſecond Column, under Angles, in a Line with Station, No. 2; and in the ſame Line, under the Title of Diſtances in the third Column, let the Meaſure of the Line BC, in Chains and Links, be inſerted: After the ſame Manner we may proceed from C to D, and thence to E; but becauſe the Angle at E, *viz.* FED, is an external Angle, after having laid one Chain from E to *h*, and to *g*, the Diſtance from *g* to *h* is meaſured, and inſerted in the Column of Angles, in a Line with Station, No. 5, and on the Side of the Field-Book againſt that Station, we make an Aſteriſk thus *, or any other Mark, to ſignify that to be an external Angle, or one meaſured out of the Ground. Proceed we then as before from E to F, to G, and thence to A, meaſuring the Angles and Diſtances, and placing them as before in the Field-Book oppoſite to their reſpective Stations; ſo will the Field-Book be compleated in manner following.

N. B. After this Manner the Angles for inacceſſible Diſtances may be taken, and the Method of conſtructing or laying them down, as well as the Conſtruction of the Map, from the following Field-Notes, muſt be obvious from the Method of taking them.

The Form of the Field-Book, with the Title.

A Field-Book of Part of the Land of *Grange*, in the Pariſh of *Portmarnock*, Barony of *Coolock*, and County of *Dublin*; being Part of the Eſtate of *L. P.* Eſq; ſet to *C. D.* Farmer. Surveyed *January* 30, 1752.

Taken by a Four-Pole Chain.

Remarks.	No. Sta.	Angles. C, L,	Diſtances. C. L.
Mr. *J. D*'s. Part of *Grange*	1	1.80	17.65
	2	1.79	18.50
Mr. *L. P*'s. Part of *Portmarnock*	3	1.76	28.00
Strand	4	1.41½	20.00
*	5	1.87½	14.83
Widow *J. G*'s. Part of *Grange*	6	1.14	19.41
	7	1.89	24.53

Cloſe at the 1ſt Station.

The Signification of the Remarks.

Mr. *J. D*'s Part of *Grange* mears, or is adjacent to the ſurveyed Land from the firſt to the third Station: Mr. *L. P*'s Part of *Portmarnock* mears it from the third to the fourth Station; the *Strand* then is the Mearing from thence to the ſixth, and from the ſixth to the firſt Station, the Widow *J. G*'s Part of *Grange* is the Mearing.

It is abſolutely neceſſary to inſert the Perſons Names, and Town-Lands, Strands, Rivers, Bogs, Rivulets, *&c.* which mear or circumſcribe the Land which is ſurveyed; for theſe muſt be expreſſed in the Map.

In a Survey of a Town-Land, or Eſtate, it is ſufficient to mention only the circumjacent Town-Lands, without the Occupiers Names; but when a Part only of a Town-Land is ſurveyed, then it is neceſſary to inſert the Perſon or Perſons Names, who hold any particular Parcel or Parcels of ſuch Town-Land, as mear the Parts ſurveyed.

When an Angle is very obtuſe, as moſt in our preſent Figure are, *viz.* the Angles at A, B, C, E and G; it will be beſt to lay a Chain from the angular Point as A, on each of the containing Sides to *c* and to *d*; and any where nearly in the Middle of the Angle as at *e*: meaſuring the Diſtances *ce* and *ed*; and theſe may be placed for the Angle in the Field-Book. Thus,

No.	Sta.	Angle. C. L.	Diſt. C. L.
1.		1.03 1.09 }	17.65

For when an Angle is very obtuſe, the Chord Line, as *cd*, will be nearly equal to the Radii A*c*, and A*d*; ſo if the Arc *ced* be ſwept, and the Chord Line *cd* be laid on it, it will be difficult to determine exactly, that Point in the Arc where *cd* cuts it: But if the Angle be taken in two Parts as *ce*, and *ed*; ſuch Chords may with Safety be laid on the Arc, and the Angle thence may be truly determined and conſtructed.

After the ſame Manner any Piece of Ground may be ſurveyed by a Two-Pole Chain.

PROB. II.

To take a Survey of a Piece of Ground from any Point within it, from whence all the Angles can be ſeen; by the Chain only.

Let

Plate VI. Fig. 6.

Let a Mark be fixed at any Point in the Ground, as at H, from whence all the Angles can be ſeen; let the Meaſures of the Lines HA, HB, HC, *&c.* be taken to every Angle of the Field from the Point H; and let thoſe be placed oppoſite to No. 1, 2, 3, 4, *&c.* in the ſecond Column of Radii: The Meaſures of the reſpective Lines of the Mearing, *viz.* AB, BC, CD, DE, *&c.* being placed in the third Column of Diſtances, will compleat the Field-Book. Thus,

Remarks.	No.	Radii. C. L.	Diſtances. C. L.
	1	20.00	17.65
	2	21.72	18.50
	3	21.74	28.00
	4	25.34	20.00
	5	17.20	14.83
	6	29.62	19.41
	7	21.20	24.53
	Cloſe at the firſt Station.		

If any Line of the Field be inacceſſible, as ſuppoſe CD to be, then by Way of Proof that the Diſtance CD is true, let the Meaſure of the Angle CHD be taken by the Line *oo*, with the Chain: If this Angle correſponds with its containing Sides, the Length of the Line DC is truly obtained, and the whole Work is truly taken.

Note, That in ſetting off an Angle it is neceſſary to uſe the largeſt Scale of Equal Parts, *viz.*

that

that of the Inch, which is diagonally divided into 100 Parts, in order that the Angle ſhould be accurately laid down; or if two Inches were thus divided for Angles, it would be the more exact; for it is by no Means neceſſary that the Angles ſhould be laid from the ſaid Scale with the ſtationary Diſtances.

PROB. III.

To take a Survey by the Chain only, when all the Angles cannot be ſeen from one Point within.

Plate VI. Fig. 7.

Let the Ground to be ſurveyed be repreſented by 1, 2, 3, 4, *&c.* Since all the Angles cannot be ſeen from one Point, let us aſſume 3 Points as A, B, C, from whence they may be ſeen; at each of which let a Mark be put, and the reſpective Sides of the Triangle be meaſured and ſet down in the Field-Book; let the Diſtances from A to 1, and from B to 1, be meaſured, and theſe will determine the Point 1; let the other Lines which flow from A, B, C, as well as the Circuit of the Ground, be then meaſured as the Figure directs; and thence the Map may be eaſily conſtructed.

There are other Methods may be uſed; as dividing the Ground into Triangles, and meaſuring the 3 Sides of each; or by meaſuring the Baſe and Perpendicular of each Triangle. But this we ſhall ſpeak of hereafter.

PROB. IV.

How to take any inacceſſible Diſtance, by the Chain only.

Plate VIII. Fig. 8.

Suppoſe AB to be the Breadth of a River, or any other inacceſſible Diſtance, which may be required.

Let a Staff or any other Object be ſet at B; draw yourſelf backward to any convenient Diſtance C, ſo that B may cover A; From B, lay off any other Diſtance by the River's Side to E, and compleat the Parallelogram EBCD: Stand at D, and cauſe a Mark to be ſet at F, in the Direction of A; meaſure the Diſtance in Links from E to F, and FB will be alſo given. Wherefore EF : ED : : FB : AB. Since it's plain (from Part 2. Theo. 3. Sect. 1, and Theo. 2. Sect. 1.) the Triangles EFD, BFA, are mutually equi-angular.

If Part of the Chain be drawn from B to C, and the other Part from B to E; and if the Ends at E and C be kept faſt, it will be eaſy to turn the Chain over to D, ſo as to compleat a Parallelogram; by reckoning off the ſame Number of Links you had in BC, from E to D, and pulling each Part ſtreight.

OF THE

CIRCUMFERENTOR.

THIS Inſtrument is compoſed of a Braſs circular Box, about five or ſix Inches in Diameter; within which is a Braſs Ring, divided on the Top into 360 Degrees, and numbered 10, 20, 30, *&c.* to 360: In the Center of the Box is fixed a Steel Pin finely pointed, called a Center-pin, on which is placed a Needle touched by a Loadſtone, which always retains the ſame Situation; that is, it always points to the North and South Points of the Horizon nearly, when the Inſtrument is horizontal, and the Needle at reſt.

The Box is covered with a Glaſs Lid, in a Braſs Rim, to prevent the Needle being diſturbed by Wind or Rain, at the Time of Surveying: There is alſo a Braſs Lid, or Cover, which is laid over the former to preſerve the Glaſs in carrying the Inſtrument.

This Box is fixed, by Screws, to a Braſs Index, or Ruler, of about 14 or 15 Inches in Length, to the Ends whereof are fixed Braſs Sights, which are ſcrewed to the Index, and ſtand perpendicular thereto: In each Sight is a large and a ſmall Aperture, or ſlit, one over the other; but theſe are changed, that is, if the large Aperture be uppermoſt in the one Sight, it will be loweſt in the other, and ſo of the ſmall ones: Therefore the ſmall Aperture in one is oppoſite to the large one in the other; in the Middle of which laſt, there is placed a Horſe-hair, or fine Silk Thread.

The Inſtrument is then fixed on a Ball and Socket; by the Help of which and a Screw, you can readily fix it horizontally in any given Direction; the Socket being fixed on the Head of a three-legged Staff, whoſe Legs when extended ſupport the Inſtrument, whilſt it is uſed.

How to take Field-Notes by the Circumferentor.

Plate VI. Fig. 6.

Let your Inſtrument be fixed at any Angle as A, your firſt Station; and let a Perſon ſtand at the next Angle B, or cauſe a Staff, with a white Sheet to be ſet there perpendicularly for an Object to take your View to: Then having placed your Inſtrument horizontal (which is eaſily done by turning the Box ſo, that the Ends of the Needle may be equidiſtant from it's Bottom, and it traverſes or plays freely) turn the *Flower-de-Luce* or North Part of the Box to your Eye, and looking through the ſmall Aperture, turn the Index about, 'till you cut the Perſon or object in the next Angle B, with the Horſe hair or Thread of the oppoſite Sight: The Degrees then cut by the South End of the Needle, will give the Number to be placed in the ſecond Column of your Field-Book in a Line with Station, No. 1, and expreſſes the Number of Degrees the ſtationary Line is from the North, counting quite round with the Sun.

Moſt Needles are pointed at the South End, and have a ſmall Ring at the North: Such Needles are better than thoſe which are pointed at each End, becauſe the Surveyor cannot miſtake by counting to a wrong End; which Error may be frequently committed, in uſing a two-pointed Needle.

Two-pointed Needles have ſometimes a Ring, but more uſually a Croſs towards the North End; and the South End is generally bearded towards it's Extremity, and ſometimes not, but it's Arm is a naked right Line from the Cap at the Center.

Having taken the Degrees or Bearing of the firſt ſtationary Line AB, let the Line be meaſured, and the Length thereof in Chains and Links be inſerted in the third Column of your Field-Book, under the Title of Diſtances, oppoſite to Station, No. 1.

It is cuſtomary, and even neceſſary, to cauſe a Sod to be dug up at each Station, or Place where you fix the Inſtrument; to the end, that if any Error ſhould ariſe in the Field-Book, it may be the more readily adjuſted and corrected, by trying over the former Bearings and ſtationary Diſtances.

Having done with your firſt Station, ſet the Inſtrument over the Hole or Spot where your Object ſtood, as at B, for your ſecond Station, and ſend him forward to the next Angle of the Field, as at C; and having placed the Inſtrument in an horizontal Direction, with the Sights directed to the Object at C, and the North of the Box next your Eye, count your Degrees to the South End of the Needle, which regiſter in your Field-Book, in the ſecond Column oppoſite to Station, No. 2; then meaſure the ſtationary Diſtance BC, which inſert in the third Column, and thus proceed from Angle to Angle, ſending your Object before you, 'till you return to the Place where you began, and you will have the Field-Book compleat; obſerving always to ſignify the Parties Names who hold the contiguous Lands, and the Names of the Town-Lands, Rivers, Roads, Bogs, Loughs or Leaks, *&c.* That mear the Land you ſurvey, as before; and this is the Manner of taking Field Notes by what are called Fore-Sights.

But the generality of Mearſmen frequently ſet themſelves in diſadvantageous Places, ſo as often to occaſion two or more Stations to be made, where one may do, which creates much Trouble and Loſs of Time: We will therefore ſhew how this may be remedied, by taking of Back-Sights, thus: Let your Object ſtand at the Point where you begin your Survey, as at A; leaving him there, proceed to your next Angle B, where fix your Inſtrument ſo, that you may have the longeſt View poſſible towards C. Having ſet the Inſtrument in an horizontal Poſition, turn the South Part of the Box next your Eye, and having cut your Object at A, reckon the Degrees to the South Point of the Needle, which will be the ſame as if they were taken from the Object to the Inſtrument, the Direction of the Index being the ſame. Let the Degree be inſerted in the Field-Book, and the ſtationary Diſtance be meaſured and annexed thereto, in it's proper Column; and thus proceed from Station to Station, leaving your Object in the laſt Point you left, 'till you return to the firſt Station A.

By this Method your Stations are laid out to the beſt Advantage, and two Men may do the Buſineſs of three, for one of thoſe who Chain may be your Object; but in Fore-Sights, you muſt have an Object before you, beſides two Chainmen.

It was ſaid before, that a Surveyor ſhould have a Perſon with him to carry the hinder End of the Chain, on whom he can depend: This Perſon ſhould be expert and ready at taking Off ſets, as well as exact in giving a faithful Return of the Length of every ſtationary Line. One who has ſuch a Perſon, and who uſes Back-Sights, will be able to go over near double the Ground he could at the ſame Time, by taking Fore-Sights. But if a Surveyor has no ſuch Perſon on whom he can with Safety depend, he muſt take Fore-Sights, becauſe of overſeeing the Chaining; for ſhould he take Back-Sights, he muſt be obliged, after taking his Degree, to go back to the foregoing Station, to overſee the Chaining, and by this Means to walk three Times over every Line, which is a Slavery not to be borne.

Or a Back and a Fore-Sight may be taken at one Station, thus; with the South of the Box to your Eye, obſerve from B the Object A, and ſet down the Degree in your Field-Book, cut by the South End of the Needle. Again, from B obſerve an Object at C, with the North of the Box to your Eye, and ſet down the Degree cut by the South Point of the Needle, ſo have you the Bearings of the Lines AB and BC; you may then ſet up your Inſtrument at D, from whence take a Back-Sight to C, and a Fore-Sight to E; thus the Bearings may be taken quite round, and the ſtationary Diſtances being annexed to them, will compleat the Field-Book.

But in this laſt Method, Care muſt be taken to ſee that the Sights have not the leaſt Caſt on either Side; if they have, it will deſtroy all: And yet with the ſame Sights you may take a Survey by Fore-Sights, or by Back-Sights only, with as great Truth as if the Sights were ever ſo erect, provided the ſame Caſt continues without any Alteration: But upon the whole, Back-Sights only will be found the readieſt Method.

If your Needle be pointed at each End, in taking Fore-Sights, you may turn the North Part of the Box to your Eye, and count your Degrees to the South Part of the Needle, as before; or you may turn the South of the Box to your Eye, and count your Degrees to the North End of the Needle.

But in Back-Sights you may turn the North of the Box to your Eye, and count your Degrees to the North Point of the Needle; or you may turn the South of the Box to your Eye, and count your Degrees to the South End of the Needle.

The Braſs Ring in the Box is divided on the Side into 360 Degrees, thus; from the North to the Eaſt into 90, from the North to the Weſt into 90, from the South to the Eaſt into 90, and from the South to the Weſt into 90 Degrees; ſo the Degrees are numbered from the North to the Eaſt or Weſt, and from the South to the Eaſt or Weſt.

The Manner of uſing this Part of the Inſtrument is this: Having directed your Sights to the Object, whether Fore or Back, as before; obſerve the two Cardinal Points of your Compaſs the Point of the

Needle lies between, (the North, South, Eaſt and Weſt being called the four Cardinal Points, and are graved on the Bottom of the Box,) putting down thoſe Points together by their initial Letters, and thereto annexing the Number of Degrees, counting from the North or South, as before, thus: If the Point of your Needle lies between the North and Eaſt, North and Weſt, South and Eaſt, or South and Weſt Points in the Bottom of the Box, then put down NE, NW, SE, or SW, annexing thereto the Number of Degrees cut by the Needle on the Side of the Ring, counting from the North or South, as before.

But if the Needle points exactly to the North, South, Eaſt, or Weſt, you are then to write down N, S, E, or W, without annexing any Degree.

This is the Manner of taking Field-Notes, whereby the Content of Ground may be univerſally determined by Calculation; and they are ſaid to be taken by the Quarter'd Compaſs, or by the four Nineties.

To find the Number of Degrees, contained in any given Angle.

Set up your Inſtrument at the Angular Point, and thence direct the Sights along each Leg of the Angle, and note down their reſpective Bearings as before; the Difference of theſe Bearings, if leſs than 180, will be the Quantity of Degrees contained in the given Angle; but if more, take it from 360, and the Remainder will be the Degrees contained in the given Angle.

O F

OF THE

THEODOLITE.

THIS Inſtrument is a Circle, commonly of Braſs, of ten or twelve Inches in Diameter, whoſe Limb is divided into 360 Degrees, and thoſe again are ſubdivided into ſmaller Parts as the Magnitude of it will admit; ſometimes by equal Diviſions, and ſometimes by Diagonals, drawn from one concentric Circle of the Limb to another.

In the Middle is fixed a Circumferentor, with a Needle; but this is of little or no Uſe, except in finding a Meridian Line, or the proper Situation of the Land.

Over the Braſs Circle is a Pair of Sights, fixed to a moveable Index, which turns on the Center of the Inſtrument, and upon which the Circumferentor Box is placed.

This Inſtrument will either give the Angles of the Field, or the Bearing of every ſtationary Diſtance Line, from the Meridian; as the Circumferentor and Quartered Compaſs do.

First then, *To take the Angles of the Field.*

Plate VI. Fig. 6.

Lay the Ends of your Index to 360, and 180;

turn the whole about with the 360 from you, direct the Sights from A to G, and ſcrew the Inſtrument faſt; direct them from A, to cut the Object at B; the Degree then cut by that End of the Index which is oppoſite to you, will be the Quantity of the Angle GAB, to place in your Field-Book; to which annex the Meaſure of the Line AB, in Chains and Links: Set up your Inſtrument at B, unſcrew it, and lay the Ends of your Index to 360, and 180; turn the whole about with the 360 from you, or 180 next you, 'till you cut the Object at A; ſcrew the Inſtrument faſt, and direct your Sights to the Object at C, and the Degree then cut by that End of the Index which is oppoſite to you, will be the Quantity of the Angle ABC. Thus proceed from Station to Station, ſtill laying the Index to 360, turning it from you, and obſerving the Object at the foregoing Station, ſcrewing the Inſtrument faſt, and obſerving the Object at the following Station, and counting the Degrees to the oppoſite End of the Index, will give you the Quantity of each reſpective Angle.

LEMMA.

All the Angles of any Polygon, are equal to twice as many Right-Angles as there are Sides leſs by four. Thus, all the Angles A, B, C, D, E, F, G, are equal to twice as many Right-Angles, as there are Sides in the Figure, leſs by four.

Plate VI. Fig. 6.

Let the Polygon be diſpoſed into Triangles, by Lines drawn from any aſſigned Point H within it, as by the Lines HA, HB, HC, &c. It is evident then (by Theo 5. Sect 1.) that the three Angles of each Triangle are equal to two right; and conſequently, that the Angles in all the Triangles, are twice as many right ones as there are Sides: But all the Angles about the Point H, are equal to four right (by Cor. 2. Theo. 1. Sect. 1.); therefore the remaining Angles are equal to twice as many right ones as there are Sides in the Figure, abating four. Q. E. D.

SCHOLIUM.

Hence we may know if the Angles of a Survey be truly taken: For if their Sum be equal to twice as many Right Angles, as there are Stations, abating four Right-Angles, you may conclude that the Angles were truly taken, otherwiſe not.

If you take the Bearing of any Line with the Circumferentor, that Bearing will be the Number of Degrees the Line is from the North; conſequently the North muſt be a like Number of Degrees from the Line, and thus the North, and of courſe the South, as well as the Eaſt and Weſt, or the Situation of the Land, is obtained.

Secondly, *To take the Bearing of each reſpective Line from the Meridian; or to perform the Office of the Circumferentor, or Quarter'd Compaſs, by the Theodolite.*

Set your Inſtrument at the firſt Station, and lay the Index to 360 and 180, with the *Flower-de Luce* of the Box next the 360; unſcrew the Inſtrument, and turn the whole about, 'till the North and South Points of the Needle cut the North and South Points in the Box; then ſcrew it faſt, ſo is the Inſtrument

North, and South, abſtracted of the Variation.

The Circumferentor Box may be then taken off.

Direct the Sights to the Object at the ſecond Station, and the Degree cut by the oppoſite End of the Index willl be the Bearing of that Line from the North, and the ſame that the Circumferentor would give.

After having meaſured the ſtationary Diſtance, ſet up your Inſtrument at the ſecond Station; unſcrew it, and ſet either End of the Index to the Degree of the laſt Line, and turning the whole about with that Degree towards you, direct your Sights to an Object at the foregoing Station, and ſcrew the Inſtrument faſt; it will then be parallel to it's former Situation, and conſequently North and South: Direct then your Sights to an Object at the following Station, and the Degree cut by the oppoſite End of the Index, will be the Bearing of that Line.

In like manner you may proceed through the whole.

If the Braſs Circle be divided into the four Ninties, from 360 and 180, and the Letters N, S, E, W, be applied to them; the Bearings may be obtained by putting down the Letters the far or oppoſite End of the Index lies between, and annexing thereto the Degrees from the N or S; and this is the ſame as the Quarter'd Compaſs.

If you keep the Compaſs Box on, to ſee the mutual Agreement of the two Inſtruments; after having fixed the Theodolite North and South, as before,

before; turn the Index about with the North End, or *Flower-de-Luce*, next your Eye, and count the Degree to the oppoſite, or South End of the Index, and this will correſpond with the Degree cut by the South End of the Needle.

At the Second, or next Station, unſcrew the Inſtrument, and ſet the South of the Index to the Degree of the laſt Station; turn the whole about, with the South of the Index to you, and cut the Object at the foregoing Station; ſcrew the Inſtrument faſt, and with the North of the Index to you, cut the Object at the next following Station, the Degree then cut by the South of the Index, will correſpond with the Degree cut by the South End of the Needle, and ſo through the whole.

Some Theodolites have a ſtanding Pair of Sights fixed at 360 and 180, beſides thoſe on the moveable Index: If you would uſe both, look thro' the ſtanding Sights, with the 180 next you, to an Object at the foregoing Station; ſcrew the Inſtrument faſt, and direct the upper Sights on the moveable Index, to the Object at the following Station, and the Degree cut by the oppoſite End of the Index, will give you the Quantity of the Angle of the Field.

Two pair of Sights can be of no Uſe in finding the Angles from the Meridian; and inaſmuch as one Pair is ſufficient to find the Angles of the Field, the ſecond can be of no Uſe: Beſides, they obſtruct the free Motion of the moveable Index, and therefore are rather an Incumberance than of any real Uſe. Some will have it, that they are uſeful with the others, for ſetting off a right Angle, in taking an Off-ſet; and ſurely this is as eaſily performed by the one Pair on the moveable Index: Thus, if you lay the Index to 360, and 180, and cut the Object either in the laſt or following Station, ſcrew the Inſtrument faſt, and turn the Index to 90 and 270, and then it will be at Right Angles with the Line. So that the ſmall Sights, or thoſe to the Circle, can be of no additional Uſe to the Inſtrument, and therefore ſhould be laid aſide as uſeleſs.

This Inſtrument may be uſed in windy and rainy Weather, as well as in mountainous and hilly Grounds; for it does not require an horizontal Poſition to find the Bearing, or Angle, as the Needle doth; and therefore is preferred to any Inſtrument that is governed by the Needle.

Mr. *Gabriel Stokes*, Mathematical Inſtrument Maker, and Surveyor of Land, has contrived a ſmall Inſtrument of this Nature, of about ſix or ſeven Inches Diameter, which is accurately divided, and is much more portable than a Theodilite, which he calls a *Pantometron*: This, with all other Mathematical Inſtruments mentioned in this Book, he ſells at his Shop in *Eſſex-Street*, *Dublin*.

Of the SEMICIRCLE.

THIS Inſtrument, as its Name imports, is a half Circle, divided from it's Diameter into 180 Degrees, and from thence again, that is, from 0, to 360: It is generally made of Braſs, and is from 8 to 16 Inches Diameter.

On the Center there is a moveable Index with Sights, on which is placed a Circumferentor-Box, as in the Theodolite.

This Inſtrument may be uſed as the Theodolite in all Reſpects; but with this Difference, when you are to reckon the Degree to that End of the Index which is off of the Semicircle, you may find it at the other End, reckoning the Degree from 180 forwards.

Of the PLANE TABLE.

A PLANE TABLE is an *Oblong* of Oak, or other Wood, about 15 Inches long, and 12 broad; they are generally compoſed of 3 Boards, which are eaſily taken aſunder, or put together, for the Conveniency of Carriage.

There is a Box Frame, with 6 Joints in it, to take off and put on as Occaſion ſerves; it keeps the Table together, and is likewiſe of Uſe to keep down a Sheet of Paper which is put thereon.

The Outſide of the Frame is divided into Inches and Tenths, which ſerve for ruling Parallels or Squares on the Paper, or for ſhifting it, when Occaſion ſerves.

The Inſide of the Frame is divided into 360 Degrees, which, tho' unequal on it, yet are the Degrees of a Circle produced from it's Center, or Center of the Table, where there is a ſmall Hole.

The Degrees are ſubdivided as ſmall as their Diſtance will admit; at every tenth Degree are two Numbers, one the Number of Degrees, the other it's Complement to 360.

There is another Center Hole, about ¼ of the Table's Breadth from one Edge, and is in the Middle between the two Ends. To this Center Hole on the other Side of the Frame, there are the Diviſions of a Semicircle, or 180 Degrees; and theſe again are ſubdivided into Halfs, or Quarters, as the Size of the Inſtrument will admit.

That Side of the Frame on which the 360 Degrees are, ſupplies the Place of a Theodolite, the other that of a Semicircle.

There is a Circumferentor Box of Wood, with a Paper Chart at the Bottom, applied to one Side of the Table by a Dove-tail Joint, faſtened by a Screw. This Box (beſides its rendering the Plane Table capable of anſwering the End of a Circumferentor) is very uſeful for placing the Inſtrument in the ſame Poſition every Remove.

There is a Braſs Ruler or Index, of about two Inches broad, with a ſharp or fiducial Edge, at each End of which is a Sight: On the Ruler are Scales of Equal Parts, with and without Diagonals, and a Scale of Chords; the whole is fixed on a Ball and Socket, and ſet on the three legged Staff.

1. *To take the Angles of the Field by the Table.*

Having placed the Inſtrument at the firſt Station, turn it about 'till the North End of the Needle be over the Meridian, or *Flower-de-Luce* of the Box, and there ſcrew it faſt. Aſſign any convenient Point, to which apply the Edge of the Index, ſo as thro' the Sights you may ſee the Object in the laſt Station, and by the Edge of the Index from the Point draw a Line. Again, turn about the Index with its Edge to the ſame Point, and thro' the Sights obſerve the Object in the ſecond Station, and from the Point, by the Edge of the Index, draw another Line; ſo is the Angle laid down: On that laſt Line ſet off the Diſtance to the ſecond Station, in Chains and Links: Apply your Inſtrument to the ſecond Station, taking the Angle as before; and after the like Manner proceed 'till the whole is finiſhed.

This Method may be uſed in good Weather, if the Needle be well touched and plays freely; but if it be in windy Weather, or the Needle out of Order, it is better, after having taken the firſt Angle as before, and having removed your Inſtrument to the ſecond Station, and placed the Needle over the Meridian Line as before, to lay the Index on the laſt drawn Line, and look backward thro' the Sights, if you then ſee the Object in the firſt Station, the Table is fixed right, and the Needle is true; if not turn the Table about, the Index lying on the laſt Line, 'till through the Sights you ſee the Object in the firſt Station; and then ſcrew it faſt, and keeping the Edge of the Index to the ſecond Station, direct your Sights to the next; draw a Line by the Edge of the Index, and lay off the next Line; and proceed ſo thro' the whole without uſing the Needle, as you do with the *Theodolite.*

If the Sheet of Paper on the Table be not large enough to contain the Map of the Ground you ſurvey, you muſt put on a clean Sheet, when the other is full; and this is called Shifting of Paper, and is thus performed.

Plate VI. Fig. 8.

Let ABCD repreſent the Sheet of Paper on the Plane Table, upon which the Plot E, F, G, H, I, K, L, M, is to be drawn; let the firſt Station be E, proceed as before from thence to F, and to G; then proceeding to H, you find there is not Room on your Paper for the Line GH: However, draw as much of the Line GH, as the Paper can hold, or draw it to the Paper's Edge. Move your Inſtrument back to the firſt Station E, and proceed the contrary Way to M, and to L; but in going from thence to K, you again find your Sheet won't hold it; however, draw as much of the Line LK on the Sheet, as it can hold.

Take that Sheet off the Table, firſt obſerving the Diſtance *oo* of the Lines GH, and LK, by the Edge of the Table; take off that Sheet, and mark it with No. 1, to ſignify it to be the firſt taken off. Having then put on another Sheet, lay that Diſtance *oo* on the contrary End of the Table, and ſo proceed as before with the Reſidue of the Survey, from *o* to H, to K, and thence to *o*; ſo is your Survey compleat.

In the like Manner you may proceed to take off, and put on, as many Sheets as are convenient; and theſe may afterwards be joyned together with Mouth Glue, or fine white Water very thin.

If the Index be fixed to the firſt Center, uſing the 360 Side, it will then ſerve as a Theodolite, and when to the ſecond Center, uſing the 180 Side, it will ſerve as a Semicircle; by either of which you may ſurvey in rainy Weather, when you can't have Paper on the Table.

To take an Angle of Altitude by the *Circumferentor*, *Theodolite*, *Semicircle*, or *Plane Table*.

1. *To take an Angle of Altitude, by the Circumferentor.*

LET the Glaſs Lid be taken off, and let the Inſtrument be turned on one Side, with the Stem of the Ball into the Notch of the Socket, ſo that the Circle may be perpendicular to the Plane of the Horizon; let the Inſtrument be placed in this Situation before the Object, ſo that the Top thereof may be ſeen thro' the Sights: Let a Plummet be ſuſpended from the Center Pin, and the Object being then obſerved, the Complement of the Number of Degrees, comprehended between the Thread of the Plummet, and that Part of the Inſtrument which is next your Eye, will give the Angle of Altitude required.

2. If an Angle of Altitude is to be taken by the Theodolite, or Semicircle, let a Thread be run thro' a Hole at the Center, and a Plummet be ſuſpended by it; turn the Inſtrument on one Side, by the Help of the Ball and Notch in the Socket for that Purpoſe, ſo that the Thread may cut 90, having 360 Degrees next you: Screw it faſt in that Poſition, and thro' the Sights cut the Top of the Object;

Objects; and the Degrees then cut by the End of the Index next you, are the Degrees of Elevation required. An Angle of Depreſſion is taken the contrary Way.

3. By the Plane Table an Angle of Altitude is taken in the like Manner, by ſuſpending a Plummet from the Center thereof, having turned the Table on one Side, and fix'd the Index to the Center by a Screw,. ſo as to move freely, let the Thread cut 90, look thro' the Sights as before, and you have the Angle of Elevation, and on the contrary that of Depreſſion.

OF THE

PROTRACTOR.

THE Protractor is a Semicircle annex'd to a Scale, and is made of Braſs, Ivory, or Horn; it's Diameter is generally about five or ſix Inches.

The Semicircle contains three concentric Semicircles, at ſuch Diſtances from each other, that the Spaces between them may contain Figures.

The outward Circle is numbered from the Right to the left Hand, with 10, 20, 30, *&c.* to 180 Degrees; the middlemoſt the ſame Way, from 180, to 360 Degrees; and the innermoſt, from the upper Edge of the Scale both ways, from 10, 20, 30, *&c.* to 90 Degrees.

It is eaſy to conceive that the Protractor, tho' a Semicircle, may be made to ſupply the Place of a whole Circle; for if a Line be drawn, and the Center-hole of the Protractor be laid on any Point in that Line, the upper Edge of the Scale correſponding with that Line, the Diviſions on the Edge of the Semicircle will run from 0, to 180, from Right to Left: Again, if it be turned the other Way, or downwards, keeping the Center-hole thereof on the aforeſaid Point in the Line, then the Diviſions will run from 180 to 360, and ſo compleats an entire Circle with the former Semicircle.

The Uſe of the Protractor is to lay off Angles, and to delineate or draw a Map, or Plan, of any Ground from the Field-Notes; and is performed in the following Manner.

To protract a Field-Book, when the Angles are taken from the Meridian.

Plate VI. Fig. 9.

On your Paper, rule Lines parallel to each other, at an Inch aſunder, (being moſt uſual) or at any other convenient Diſtance; on the left End of the Parallels put N, for North, and on the right S, for South; put E at the Top, for Eaſt, and W at the Bottom of your Paper for Weſt.

Then let the following Field-Book be that which is to be protracted, the Bearings being taken from the Meridian, whether by a Circumferentor, Theodolite, or Semicircle, and meaſured with a Two-Pole Chain.

No	Bearing.	C. L
1	283½	55.20
2	348¾	12.36
3	317	29.20
4	266	55.20
5	193½	40.00
6	124	76.00
7	63¾	87.02

Cloſe at the firſt Station.

Pitch upon any convenient Point on your Paper, for your firſt Station, as at 1, on which lay the Center-hole of your Protractor, with a Protracting-Pin; then if the Degrees be leſs than 180, turn the Arc of your Protractor downwards, or towards the Weſt; but if more than 180, upwards, or towards the Eaſt.

Or if the Right Hand be made the North, and the Left the South, the Weſt will be then up, and the Eaſt down.

In this Caſe, if the Degrees be leſs than 180, turn the Arc of your Protractor upwards, or towards the Weſt; and if more, downwards, or to the Eaſt.

By the foregoing Field-Book, the firſt Bearing is 283½; turn the Arc of your Protractor upwards, keeping the Pin in the Center-hole, move the Protractor ſo that the Parallel Lines may cut oppoſite Diviſions, either on the Ends of the Scale, or on the Degrees, and then it is parallel. This muſt be always firſt done, before you lay off your Degrees.

Then by the Edge of the Semicircle keeping the Protractor ſteady, with the Pin prick the firſt Bearing 283½, and from the Center Point, thro' that Point or Prick, draw a blank Line with the Pin, on which from a Scale of equal Parts, or from the Scale's Edge of the Protractor, lay off the Diſtance 55C. 20L. ſo is that Station protracted.

At the End of the firſt Station, or at 2, which is the Beginning of the ſecond, with the Pin place the Center of the Protractor, turning the Arc up, becauſe the Bearing of the ſecond Station is more than 180, *viz.* 348¼. Place your Protractor parallel as before, and by the Edge of the Semicircle, with the Pin prick at that Degree, thro' which and the End of the foregoing Station, draw a blank Line, and on it ſet the Diſtance of that Station.

In the like Manner proceed thro' the whole, only obſerve to turn the Arc of your Protractor down, when the Degrees are leſs than 180.

If you lay off the ſtationary Diſtances by the Edge of the Protractor, it is neceſſary to obſerve, that if your Map is to be laid down by a Scale of 40 Perches to an Inch, every Diviſion on the Protractor's Edge will be one Two-Pole Chain; ½ a Diviſion will be 25 Links, and ¼ of a Diviſion will be 12½ Links.

If your Map is to be laid down by a Scale of 20 Perches to an Inch, two Diviſions will be one Two-Pole Chain; one Diviſion will be 25 Links; ½ a Diviſion 12½ Links, and ¼ of a Diviſion will be 6¼ Links.

In the general, if 25 Links be multiplied by the Number of Perches to an Inch, the Map is to be laid down by, and the Product be divided by 20; (or which is the ſame Thing, if you cut off one and take the half,) you will have the Value of one Diviſion on the Protractor's Edge, in Links and Parts.

EXAMPLE.

1. How many Links in a Diviſion, if a Map be laid down by a Scale of 8 Perches to an Inch?

25
8

2|0)20|0

---

10 Links. Anſwer.

2. How many Links in a Diviſion, if a Map be laid down by a Scale of 10 Perches to an Inch?

25
10

2|0)25|0

12.5 or 12½ Links. Anſwer.

And ſo of any other.

To protract a Field-Book, taken by the Angles of the Field.

Note, We here ſuppoſe the Land ſurveyed is kept on the right Hand as you ſurvey.

Draw a blank Line with a Ruler of a Length greater than the Diameter of the Protractor; pitch upon any convenient Point therein, to which apply the Center-hole of your Protractor with your Pin, turning the Arc upwards if the Angle be leſs than 180, and downwards if more; and obſerve to keep the upper Edge of the Scale, or 180 and 0 Degrees upon the Line: Then Prick off the Number of Degrees contained in the given Angle, and draw a Line from the firſt Point through the Point at the Degrees; upon which lay the ſtationary Diſtance. Let this Line be lengthened forwards and backwards, keeping your firſt Station to the Right, and ſecond to the Left; and lay the Center of your Protractor over the ſecond Station, with your Pin, turning the Arc upwards, if the Angle be leſs than 180, and downwards, if more; and keeping the 180 and 0 Degrees on the Line, prick off the Number of Degrees contained in the given Angle, and thro' that Point and the laſt Station draw a Line, on which lay the ſtationary Diſtance: And in like Manner proceed through the whole.

In all Protractions, if the End of the laſt Station falls exactly in the Point you begin at, the Field-Work and Protraction are truly taken, and performed; if not, an Error muſt have been committed in one of them: in ſuch Caſe, make a ſecond Protraction; if this agrees with the former, and neither meet or Cloſe, the Fault is in the Field-Work, and not in the Protraction: And then a Re-ſurvey muſt be taken.

SECT.

SECT. IV.

Containing Five various Methods by which the Areas of Right-lined Figures may be determined, two of which were never yet publiſhed.

DEFINITION.

THE Area or Content of any Plane Surface in Perches, is the Number of Square Perches, that Surface contains.

Plate VII. Fig. 1.

Let ABCD repreſent a Rectangular Parallelogram, or Oblong: Let the Side AB, or DC, contain 8 Equal Parts; and the Side AD, or BC, three of ſuch Parts: Let the Line AB be moved in the Direction of AD, 'till it has come to EF; where AE, or BF, (the Diſtance of it from it's firſt Situation) may be equal to one of the Equal Parts. Here 'tis evident, that the generated Oblong ABEF, will contain as many Squares as the Side AB contains Equal Parts, which are 8; each Square having for it's Side one of the Equal Parts, into which ~~AB, or AD, is~~ divided. Again, let AB move on 'till it comes to GH, ſo as GE, or HF, may be equal to AE, or BF; then it is plain, that the Oblong AGHB, will contain twice as many

Squares as the Side AB contains Equal Parts. After the ſame Manner it will appear, that the Oblong ADCB will contain three times as many Squares as the Side AB contains equal Parts; and in general, that every Rectangular Parallelogram, whether Square or Oblong, contains as many Squares as the Product of the Number of Equal Parts in the Baſe, multiplied into the Number of the ſame Equal Parts in the Height, contains Units, each Square having for it's Side one of the Equal Parts.

Hence ariſes the Solution of the following Problems.

PROB. I.

To find the Content of a Square Piece of Ground.

1. Multiply the Baſe in Perches, into the Perpendicular in Perches, (or ſquare the Baſe) the Product will be the Content in Perches; and becauſe 160 Perches make an Acre, it muſt thence follow, that

Any Area, or Content in Perches, being divided by 160, will give the Content in Acres; the remaining Perches, if more than 40, being divided by 40, will give the Roods, and the laſt Remainder, if any, will be Perches.

Or thus:

2. Square the Side in Four-Pole Chains and Links, and the Product will be ſquare Four-Pole Chains and Links; divide this by 10, or cut off one more than the Decimals, which are five in all, from the Right towards the Left: The Figures

reſting to the Left are Acres, becauſe 10 ſquare Four-Pole Chains make an Acre, and the remaining Figures are Decimal Parts of an Acre. Multiply the five Figures to the Right by 4, cutting 5 Figures from the Product, and if any Figure be to the Left of them, it is a Rood, or Roods; multiply the laſt cut off Figures by 40, cutting off five, or (which is the ſame thing) by 4, cutting off four; and the remaining Figures to the Left, if any, are Perches.

1. The firſt Part is plain, from conſidering that a Piece of Ground in a ſquare Form, whoſe Side is a Perch, muſt contain a Perch of Ground; and that 40 ſuch Perches make a Rood, or Stang, and four Roods an Acre; or which is the ſame Thing, that 160 ſquare Perches make an Acre, as before.

2. A ſquare Four-Pole Chain (that is a Piece of Ground four Poles or Perches every Way) muſt contain 16 ſquare Perches; and ſince 160 Perches make an Acre, therefore 10 times 16 Perches, or 10 ſquare Four-Pole Chains make an Acre.

Note, that the Chains given, or required, in any of the following Problems, are ſuppoſed Two-Pole Chains, that Chain being moſt commonly uſed in this Kingdom.

EXAMPLE.

C. L.

Let ABCD be a ſquare Field, whoſe Side is 14.29; I demand the Content in Acres.

By

C. L.
By Problem 4. Section 3. 14.29, are equal to
29.16 Perches.
29.16

17496
2916
26244
5832
A. R. P.
160)850.3056(5. 1. 10. Content.

40)50(1 Rood

10 Perches.

Or thus:

C. L. C. L.
14.29 is equal to 7. 29 of Four-Pole Chains, by
Prob. 1. Sect. 3. 7. 29

6561
1458 A. R. P.
5103 Cont. as before 5. 1. 10.

Acres 5|31441
4

Rood 1|25764
4

Perches 10|3056

It is required to lay down a Map of this Piece of Ground, by a Scale of twenty Perches to an Inch.

Take 29.16 the Perches of the given Side, from the ſmall Diagonal on the common Surveying Scale, where 20 ſmall, or two of the large Diviſions are an Inch; make a Square whoſe Side is that Length, (by Prob. 9. Sect. 1.) and it is done.

PROB. II.

To find the Side of a Square, whoſe Content is given.

Extract the Square Root of the given Content in Perches, and you have the Side in Perches, and conſequently in Chains.

EXAMPLE.

It is required to lay out a ſquare Piece of Ground which ſhall contain 12A. 3R. 16P. Required the Number of Chains in each Side of the Square; and to lay down a Map of it, by a Scale of 40 Perches to an Inch.

A. R. P.
12. 3. 16
4

51
40

C. L.
2056(45.34 Perch, which is 22. 33½, by Prob. [6. Sect. 3.

85)456

903)3100

9064)39100

To draw the Map.

From a Scale where 4 of the large, or 40 of the ſmall Diviſions are an Inch, take 45.34, the Perches of the Side, of which make a Square.

PROB. III.

To find the Content of an Oblong Piece of Ground.

Multiply the Length by the Breadth, for the Content.

EXAMPLE.

Plate I. Fig. 3.

Let ABCD be an oblong Piece of Ground, whoſe Length AB is 14C. 25L., and Breadth 8C. 37L. I demand the Content in Acres, and alſo to lay down a Map of it, by a Scale of 20 Perches to an Inch.

C. L. Perches.
14.25 = 29.00 } By Prob. 4. Sect. 3.
8.37 = 17.48 }

15732
3496

A. R. P.
160)506.9200(3. 0. 27. Content.

26 Perch, or near 27.

Or thus:

4 Pole C.
C. L. C. L.
14.25 = 7.25 } By Prob. 1. Sect. 3.
8.37 = 4.37 }

5075
2175
2900

Acres 3|16825
4

Rood |67300
4

Perches 26|9200

To draw the Map.

Make an Oblong (by Schol. to Prob. 9. Sect. 1.) whoſe Length, from a Scale of 20 to an Inch, may be 29 Perches, and Breadth, 17.48 Perches.

PROB. IV.

The Content of an Oblong Piece of Ground, and one Side given, to find the other.

Divide the Content in Perches, by the given Side in Perches, the Quotient is the required Side in Perches; and thence it may be eaſily reduced to Chains.

EXAMPLE.

EXAMPLE.

C. L.
There is a Ditch 14. 25 long, by the Side of which it is required to lay out an Oblong Piece of Ground, which ſhall contain 3A. 0R. 27P: What Breadth muſt be layed off at each End of the Ditch, to incloſe the 3A. 0R. 27P?

A. R. P.
3. 0. 27.
4

12
40
Perch. C. L.
29)507(17. 48 = 8. 37 Breadth.

217

140

240

8

The Map is done as the laſt.'

PROB. V.

To find the Content of a Piece of Ground, in Form of an Oblique Angular Parallelogram; or of a Rhombus, or Rhomboides.

Multiply the Baſe into the Perpendicular Height. The Reaſon is plain from Theo. 13. Sect. 1.

EXAMPLE.

EXAMPLE.

Plate VII. Fig. 2.

Let ABCD be a Piece of Ground in Form of a Rhombus, whoſe Baſe AB is 22 Chains, and Perpendicular DE, or FC, 20 Chains. Required the Content.

$$\begin{array}{l} \text{C.} \quad \text{C.} \\ \left.\begin{array}{r} 22 = 11.0 \\ 20 = 10.0 \end{array}\right\} \text{4 Pole Chains.} \\ \hline \text{Acres } 11|0 \end{array}$$

Or,

$$\begin{array}{l} \text{C.} \\ \left.\begin{array}{r} 22 = 44 \\ 20 = 40 \end{array}\right\} \text{Perches.} \\ \hline 160)1760(11 \text{ Acres.} \\ \hline \quad\;\; 160 \\ \hline \end{array}$$

The Converſe of this is done by Prob. 4. and the Map is drawn, by laying of the Perpendicular on that Part of the Baſe from whence it was taken: Joining the Extremity thereof to that of the Baſe, by a Right Line, and thence compleat the Parallelogram.

PROB.

PROB. VI.

To find the Content of a Triangular Piece of Ground.

Multiply the Baſe by Half the Perpendicular, or the Perpendicular by Half the Baſe; or take Half the Product of the Baſe into the Perpendicular.

The Reaſon hereof is plain, from Cor. 2. Theo. 12. Sect. 1.

EXAMPLE.

Plate I. Fig. 16.

Let ABC be a Triangular Piece of Ground, whoſe longeſt Side or Baſe BC, is 24 C. 38 L. and Perpendicular AD, let fall from the oppoſite Angle, is 13 C. 28 L. Required the Content.

```
                C.  L.    C.  L.
1.  Baſe 24. 38 = 12.38 }
         ½ Perp.   3.39 } 4 Pole Chains.
                  ------
                  11142
                  3714
                 3714
                 ------
        Acres    4|19682
                       4
                 -------
        Rood     7|8728
                     40
                 -------
     Perches    31)4912

                A.  R.  P.
    Content     4.  0.  31.
```

C. L. C. L.
Perp. 13.28 = 6. 78 } Four-Pole Chains, by Prob.
½ Perp. 6.39 = 3. 39 } 1. Sect. 3.

Or 2dly. Perp. 6.78 of Four-Pole Chains.
½ Baſe 6.19

6102
678
4068

A. R. P.
4|19682 = 4. 0. 31.

Or 3dly. Baſe 12.38 Four-Pole Chains.
Perp. 6.78

9904
8666
7428

839364

A. R. P.
It's ½ 4|19682 = 4. 0. 31.

Or the Baſe, and Perpendicular, may be reduced to Perches; and the Content may be thence obtained thus:

1. Baſe

C. L. Perches.
Perp. 13.28 = 27.12
Half the Perp. 13.56
By Prob. 4. Sect. 3.

Perches. C. L.
1. Base 49.52 = 24.38
½ Perp. 13.56

29712
24760
14856
4952

A. R. P.
160)671.4912(4. 0. 31.

31

Perch
2. Perp. 27.12
Half Base 24.76

16272
18984
10848
5424

A. R. P.
671.4912 = 4. 0. 31.

Perch.
3. Baſe 49.52
Perp. 27.12

9904
4952
34664
9904

1342.9824

A. R. P.
671.4912 = 4. 0. 31.

The Map may be readily drawn, having the Diſtance from either End of the Baſe, to the Perpendicular given; as may be evident from the Figure.

PROB. VII.

The Content of a Triangular Piece of Ground, and the Baſe given, to find the Perpendicular.

Divide the Content in Perches, by Half the Baſe in Perches; and the Quotient will give you the Perpendicular in Perches, and ſo in Chains.

EXAMPLE.

Plate I. Fig. 16.

Let BC be a Ditch, whoſe Length is 24C. 40L. by which it is required to lay out a Triangular Piece of Ground, whoſe Content ſhall be 4A. 1R. 10P. Required the Perpendicular.

4. 1. 10.

C. L. Perch.
Baſe 24.40 = 49.6
Half the Baſe = 24.8

A. R. P.
4. 1. 10.
4
——
17
40
—— Perches.
24.8)690(27.82
——
1940
——
2040
——
560
——
64
——

Perch. C. L.
Anſwer Perp. 27.82 = 13.45

This Perpendicular being laid on any Part of the Baſe, and Lines run from it's Extremity to the Ends of the Baſe, will lay out the Triangle (by Cor. to Theo. 13. Sect. 1.) ſo that the Perpendicular may be ſet on that Part of the Baſe, which is moſt convenient and agreeable to the Parties concerned.

LEMMA.

If from half the Sum of the Sides of any Plane Triangle ABC, each particular Side be taken; and if the half Sum, and the three Remainders be multiplied continually into each other, the Square Root of this Product will be the Area of the Triangle.

Plate VIII. Fig. 9.

Biſect any two of the Angles, as A and B, with the Lines AD, BD, meeting in D; draw the Perpendiculars DE, DF, DG.

The Triangle AFD is equiangular to AED; for the Angle FAD = EAD by Conſtruction, and AFD = AED, being each a right Angle, and of Conſequence ADF = ADE; wherefore AD : DF : : AD : DE: And ſince AD bears the ſame Proportion to DF, that it doth to DE, DF = DE, and the Triangle AFD = AED. The ſame way DE = DG, and the Triangle DEB = DGB, and FD = DE = DG; therefore D will be the Center of a Circle that will paſs through E, F, G.

In the ſame Way if A and C were biſected, the ſame Point D would be had; therefore a Line from D to C will biſect C, and thus the Triangles DFC DGC will be alſo equal.

Produce CA to H, 'till AH = EB or GB; ſo will HC be equal to half the Sum of the Sides, *viz.* to $\frac{1}{2}$ AB, + $\frac{1}{2}$ AC + $\frac{1}{2}$ BC: For FC, FA, EB, are ſeverally equal to CG, AE, BG; and all theſe together are equal to the Sum of the Sides of the Triangle; therefore FC + FA + EB or CH, are equal to half the Sum of the Sides.

FC = CH—AB, for AF = AE, and HA—EB; therefore HF = AB: And AF = CH—BC; for CF = CG, and AH = GB; therefore BC = HA+FC, and AH = CH—AC.

Continue DC, 'till it meets a Perpendicular drawn upon H, in K; and from K draw the Perpendicular KI, and join AK.

Becauſe the Angles AHK and AIK are two right ones, the Angles HAI and K together, are equal to two right; ſince the Angles of the two Triangles contain four right: In the ſame Way FDE+FAE = (2 right Angles =) FAE+IAH; let FAE be taken from both, then FDE = IAH, and of courſe FAE = K: The quadrilateral Figures AFDE, and KHAI, are therefore ſimilar, and have the Sides about the equal Angles proportional; and it is plain the Triangles CFD and CHK are alſo proportional: Hence,

FD : HA : : FA : HK
FD : FC : : HK : HC

Wherefore by multiplying the Extreams. and Means in both, it will be, the Square of FDxHK x HC = FC x FA x HA x HK; let KH be taken from both, and multiply each Side by CH; then the Square of CH x by the Square of FD = FC x FA x HA x CH.

It is plain, by the foregoing Problem that $\frac{1}{2}$ AB x DE, + $\frac{1}{2}$ BC x DG + $\frac{1}{2}$ AC x FD = the Area of the Triangle; or that Half the Sum of the Sides, *viz.* CH x FD = the Triangle; wherefore the Square of CH x by the Square of FD = FC x

FA x HA x CH, that is, the half Sum multiplied continually into the Differences between the half Sum and each Side, will be the Square of the Area of the Triangle, and it's Root the Area. Q. E D.

Hence the following Problem will be evident.

PROB. VIII.

The Sides of any plain Triangle given to find the Area.

Plate VIII. Fig. 9.

Add the Sides, together, and from the half Sum take each Side ſeverally: Multiply the half Sum and three Remainders continually into each other, and the Square-Root of their Product will be the Area.

If the Triangle be meaſured by a Two-Pole Chain, reduce the Sides to Four-Pole Chains; and let them ſtand thus:

		C. L.	
Given	AB	10.64	the Area required.
	AC	12.28	
	BC	9.00	
	Sum	31.92	
	Half Sum	15.96	

C. L. C. L.
15.96—10.64=5.32 firſt Remainder.
15.96—12.28=3.68 ſecond Remainder.
15.96— 9.00=6.96 third Remainder.

C. L.
15.96 half the Sum.
5.32 firſt Remainder.

3192
4788
7980

84.9072
3.68 ſecond Remainder.

6792576
5094432
2547216

312.458496
6.96 third Remainder.

1874750976
2812126464
1874750976

2174.71013216

2174.

```
         2174.71013216(46.6337
         16
86     | 574
       | 516
926    | 5871
       | 5556
9323   | 31501
       | 27969
93263  | 353232
       | 279789
932667 | 7344316
       | 6528669
           815647
```

Area in ſquare Four-Pole Chains,	46.6337
Acres	4.66337
	4
Roods	2.65348
	40
Perches	26.13920

A. R. P.
Anſwer 4. 2. 26.

Or if you reduce each Side into Perches, the Operation is performed in the like Manner; and the Square Root will be Perches.

To

To perform this by Logarithms.

Inaſmuch as the Addition of Logarithms anſwers the Multiplication of their correſponding Numbers; and that the Number anſwering to the half of a Logarithm, will give the Square Root of the Number of that Logarithm, it follows,

That Half the Sum of the Logarithms of Half the Sum of the Sides, and of the three Remainders, will give the Area. Thus,

Half Sum	15.96	1.20303
Firſt Remainder	5.32	0.72591
2d Remainder	3.68	0.56585
3d Remainder	6.96	0.84261
		3.33740
Square 4 Pole Chains	46.63	1.66870
Acres	4.663	
	4	
Roods	2.652	
	4	
Perches	26.08	

	A.	R.	P.
Anſwer, as before	4.	2.	26.

If therefore you reduce a Piece of Ground into Triangles, (as that in Plate VII. Fig. 4.) and meaſure the three Sides of each; you may thus find the Content of the whole, by natural Numbers, or more readily by Logarithms.

The Figure may alſo be eaſily conſtructed by Prob. 1. Sect. 1.

PROB. IX.

To find the Content of a Trapezium Piece of Ground.

Multiply the Diagonal, or Baſe, by Half the Sum of the two Perpendiculars, or the two Perpendiculars by Half the Baſe; or take Half a Product of the Baſe into the Sum of the two Perpendiculars.

DEFINITION.

A Line contained between the two remoteſt Angles, is the Diagonal.

EXAMPLE.

Plate VII. Fig. 3.

Let ABCD be a Piece of Ground, in Form of a Trapezium, whoſe Diagonal, or Baſe, AC, is 32C. 10L; it's Perpendicular bB on one Side of that Line is 6C. 41 L. and Perpendicular dD on the other Side is 13 C. 31 L. Required the Content.

Perp.

	C. L.		C. L.	
Perp.	6.41	=	3.41	} 4 Pole Chains.
Perp.	13.31	=	6.81	
	Sum		10.22	
	Half Sum		5.11	

	C. L.		C. L.
Baſe	32.10	=	16.10
			5.11
			1610
			1610
			8050
	Acres		8|22710
			4
	Roods		|9084
			4
	Perches		36|336

The other two Methods are obvious, from what has been laid down (in the Triangle, Prob. 6.) and will produce a like Content. If both the Baſe and the Sum of the two Perpendiculars are odd, uſe the laſt Method of the three.

The Map may be drawn, by knowing in what Parts of the Diagonal the Perpendiculars were taken, and thence ſetting them off, and joining their Extremities to thoſe of the Diagonal.

The Area of the Square, Oblong, Rhombus, or Rhomboides, may be obtained in the like Manner, by a Diagonal and Perpendiculars.

If a Map of any of the foregoing Figures be given, with the Scale by which it was laid down, take the Dimenſions from that Scale of Perches, and thereby adjuſt their Contents, without reducing them to Chains.

From this, and the ſixth Problem, this following Method will be plain.

Firſt Method to find the Area of Ground.

How the Area of a Piece of Ground be it ever ſo irregular may be obtained, by dividing it into Triangles, and Trapezia.

Plate VII. Fig 4.

We here admit the Survey to be taken, and protracted; by having therefore the Map, and knowing the Scale by which it was laid down, the Content may be thus obtained.

Diſpoſe the given Map into Triangles, by fine pencilled Lines, ſuch as are here repreſented by pop'd Lines in the Scheme, and number the Triangles with 1, 2, 3, 4, *&c.* Your Map being thus prepared, rule a Table with four Columns; the firſt of which is for the Number of the Triangle, the ſecond for the Baſe of it, the third for the Perpendicular, and the fourth for the Content in Perches.

Then proceed to meaſure the Baſe of Number 1, from the Scale of Perches the Map was laid down, and place that in the ſecond Column of the Table, under the Word Baſe; and from the Angle oppoſite to the Baſe, open your Compaſſes ſo, as when one Foot is in the Angular Point, the other being moved backwards and forwards, may juſt touch the Baſe Line, and neither go the leaſt above or beneath it; that Diſtance in the Compaſſes, meaſured from the ſame Scale, is the Length of that Perpendicular, which place in the third Column, under the Word Perpendicular.

If the Perpendiculars of two Triangles fall on one and the ſame Baſe, it is unneceſſary to put down the Baſe twice, but inſert the ſecond Perpendicular oppoſite to the Number of the Triangle in the Table, and join it with the other Perpendicular by a Brace, as No. 1 & 2, 4 & 5, 6 & 7, 9 & 10, *&c.*

Proceed after this Manner, 'till you have meaſured all the Triangles; and then by Prob. 6, find the Content in Perches of each reſpective Triangle, which ſeverally place in the Table oppoſite to the Number of the Triangle, in the fourth Column, under the Word Content.

But where two Perpendiculars are joined together in the Table, by a Brace, having both one and the ſame Baſe; find the Content of each, (being a Trapezium) in Perches, by Prob. 9, which place oppoſite the Middle of thoſe Perpendiculars, in the fourth Column, under the Word Content.

Having thus obtained the Content of each reſpective Triangle and Trapezium, which the Map contains, add them all together, and their Sum will be the Content of the Map in Perches; which being divided by 160, gives the Content in Acres Thus. For

EXAMPLE.

No.	Baſe	Perp.	Content
1	24.8	17.0	412.92
2		16.3	
3	28.2	16.0	225.6
4	39.8	19.6	712.42
5		16.2	
6	49.4	29.0	1086.8
7		15.0	
8	38.7	6.7	129.64
9	40.0	17.0	600.
10		13.0	
11	42.8	10.2	481.5
12		12.3	
13	26.2	17.9	234.49
14	24.0	11.6	259.2
15		10.0	

Content in Perches 4142.57

This being divided by 160, will give 25 A. 3 R. 22 P. the Content of the Map.

```
1) 17.0
2) 16.3
   ----
   33.3
   12.4
   ----
   1332
   666
   333
  ------
  412.92
  ------

6) 29.
7) 15.
   ---
   44.
   ---
   22.
   49.4
   ----
   988
  988
  ------
  1086.8
  ------

11) 10.2
12) 12.3
    ----
    22.5
    21.4
    ----
    900
   225
  450
  ------
  481.50

3) 28.2
    8.
  -----
  225.6

4) 19.6
5) 16.2
   ----
   35.8
   19.9
   ----
   3222
  3222
  358
  ------
  712.42
  ------

8) 38.7
    6.7
   ----
   2709
  2322
  ------
  259.29
  ------
  129.64
  ------

13) 17.9
    13.1
    ----
     179
    537
   179
  ------
  234.49
  ------

9) 17.
10) 13.
    ---
    30.
    20.
    ---
    600
    ---

14) 11.6
15) 10.0
    ----
    21.6
    12.
    -----
    259.2
```

Let your Map be laid down by the largeſt Scale your Paper will admit, for then the Baſes and Perpendiculars can be meaſured with greater Accuracy than when laid down by a ſmaller Scale; and if poſſible meaſure from Scales divided diagonally.

If the Baſes and Perpendiculars were meaſured by Four Pole Chains, the Content of every Triangle, and Trapezium, may be had as before, in Problems 6 and 9, and conſequently the whole Content of the Map.

If any Part of your Map has ſhort or crooked Bounds, as thoſe repreſented in Plate VII, Fig. 5, then by the ſtreight Edge of a tranſparent Horn, draw a fine pencilled Line as AB, to ballance the Parts taken in and left out, as alſo another, BC: Theſe Parts when ſmall, may be ballanced very nearly by the Eye, or they may be more accurately ballanced by Method the Third. Join the Points A and C by a Line, ſo will the Content of the Triangle ABC, be equal to that contained between the Line AC, and the crooked Mearing from A to B, and to C: By this Method the Number of Triangles will be greatly leſſened, and the Content become more certain; for the fewer Operations you have, the leſs ſubject will you be to err; and if an Error be committed, the ſooner it may be diſcovered.

The Lines of the Map ſhould be drawn ſmall, and neat, as well as the Baſes; the Compaſſes neatly pointed, and Scale accurately divided; without ll which, you may err greatly. The Multiplications ſhould be run over twice at leaſt, as alſo the Addition of the Column Content.

From what has been ſaid, it will be eaſy to ſurvey a Field, by reducing it into Triangles, and meaſuring the Baſes and Perpendiculars by the Chain. To aſcertain the Content only, it is not material to know at what Part of the Baſe the Perpendicular was taken; ſince it has been ſhewn (in Cor. to Theo. 13. Sect. 1.) that Triangles on the ſame Baſe, and between the ſame Parallels are equal: But if you would draw a Map from the Baſes and Perpendiculars, it is evident that you muſt know at what Part of the Baſe the Perpendicular was taken, in order to ſet it off in it's due Poſition; and hence the Map is eaſily conſtructed.

Second Method to find the Area of Ground.

To determine the Area of a Piece of Ground, from the Map thereof, by the Square Inch Horn.

This Horn is tranſparent, and has on it a Square whoſe Side is an Inch; each Side of this is divided into 10 equal Parts, and the oppoſite Diviſions are joined by Right Lines, ſo the whole Square contains 10 times 10, or 100 ſmall Squares.

Let the Parallels, by which your Map is to be protracted, be an Inch aſunder, and at Right Angles to theſe, let other Parallels be drawn an Inch aſunder alſo; ſo will your Paper be divided into Square Inches. But great Care muſt be taken that the Parallels be exactly an Inch aſunder, and that the croſs Parallels be truly perpendicular to them, and alſo duly an Inch from each other.

When all your Parallels are drawn, lay the Edge of a ſtraight Ruler diagonal wiſe, on the oppoſite Angles of a Square: Then if the Ruler's Edge cuts exactly the oppoſite Angles of every Square it paſſes over, and the Diagonals or Diameters of each be equal, the Paper is truly ſquared, otherwiſe not. And if the Parallels are an Inch aſunder exactly, they will be all ſquare Inches.

Having thus prepared your Paper, and on it either protracted or drawn your Map; lay your Inch Horn on every Square thro' which any of the Lines of your Map paſs, and count the Number of ſmall Squares on your Horn which lie within the Lines of the Map, which Number place down in that Part of a Square; do the like in every Part of a Square, and in every whole Square put down 100, or Number them 1, 2, 3, *&c.* Add all thoſe Numbers which you have in Parts of Squares and whole Squares together; and their Sum will be the Number of ſmall Squares which the Map contains.

It is beſt to reckon the Parts of Squares without the Map as well as thoſe within, and by ſo doing, if the Parts without and within be equal to 100, you may reaſonably conclude that you have counted and eſtimated true.

Let ABCD, Plate VII. Fig. 7. be a Map, whoſe Area is required: Let the Paper be ſquared as before taught, and let every of the Squares in the

Scheme repreſent ſquare Inches. Let the Inch Horn, Plate VII. Fig. 6, be applied on every Square through which any of the Lines of the Map run, and count the Number of ſmall Squares contained within the Lines of the Map, which put down on your Map, as in the Scheme: Then number the whole Squares ſeparately, and add the ſmall Squares contained in them, to the Number of ſmall Squares in the Parts through which the Lines of the Map paſs, and you have the Number of ſmall Squares contained in the Map.

	61½	
25	2	
61	11	10
20	97	78¼
21	99	35
96½	69	27
71	27	28
31	98	3
98¼	28	47
22	28	52
31	11	4
476¾	531½	284¼

476¾	
531½	
284¼	
1292½	ſmall Squares in Parts of large ones.
1900	ſmall Squares in 19 large ones.
3192½	Content of the Map in ſmall Squares.

Having obtained the Number of ſmall Squares contained in your Map, you are next to conſider what Scale it was laid down by; and thence you may know what the Content of every ſmall Square is, and of courſe the Content of them all, or the Content of the Map. Thus;

If your Map be laid down by a Scale of 8 Perch to an Inch, the Side of the Square Inch will be 8, and it's Content 8 Times 8, or 64 Perches; and becauſe there are 100 ſmall Squares in a large one, the Content of every ſmall Square will be .64 of a Perch; wherefore the Number of ſmall Squares being multiplied by .64 will give their Content in Perches, and thoſe divided by 160, give the Content in Acres.

If your Map be laid down by a Scale of 10 Perches to an Inch, the Content of every Square Inch will be ten Times 10, or 100 Perches; and ſince every Square Inch contains 100 ſmall Squares, therefore every ſmall Square will be 1 Perch; conſequently the Number of ſmall Squares contained in your Map, will be the Content of it in Perches.

If your Map be laid down by 15 Perch to an Inch, a Square Inch will contain 15 Times 15, or 225 Perches; and a ſmall Square being the hundredth Part of a large one, will contain 2.25 Perches; wherefore the Number of ſmall Squares being multiplied by 2.25 will give the Content in Perches; or to the double of the Number of ſmall Squares add ¼ of them, the Sum is the Content in Perches.

If your Map be laid down by 20 Perches to an Inch, the Square Inch will then contain 20 Times 20, or 400 Perches; and 1 ſmall Square will contain 4 Perches; wherefore the Number of ſmall Squares being multiplied by 4, gives the Content in Perches.

Or divide the Sum of the ſmall Squares, having cut off the laſt Figure, by 4; the Quotient is Acres, and what remains, if any is a Rood or Roods: Then multiply your laſt Figure by 4, adding 1 for a Quarter, 2 for Half, or 3 for ¾ of a Square, (if you have any ſuch in the Sum of your ſmall Squares) and you will have the Perches. Thus the Number of ſmall Squares in the foregoing Map being 3192½, I demand the Content in Acres.

$$\frac{319|2\tfrac{1}{2}}{\text{Content } 79 : 3 : 10}$$

Cut off the laſt Figure 2, and divide 319 by 4, the Quotient is 79 Acres, and the Remainder 3, is 3 Roods. Multiply the laſt Figure 2 by 4, and add in 2 for the Half Square, and you have 10 Perches.

The Reaſon hereof is plain, when we conſider that any Number multiplied by 4, and divided by 160, will give in the Quotient the ſame, as if that Number was only divided by 40; or which is the ſame thing, cut off the laſt Figure for the 0, and divide by 4.

If your Map be laid down dy 25 Perches to an Inch, every Square Inch will be 25 Times 25, or 625 Perches; wherefore every ſmall Square will be 6.25 Perches. The Number of ſmall Squares therefore

fore being multiplied by 6, adding ¼ of their Number to the Product, will give the Content in Perches.

If your Map be laid down by a Scale of 30 Perches to an Inch, every Square Inch will contain 900 Perches, and every ſmall Square 9 Perches; the Number of ſmall Squares therefore being multiplied by 9, will give the Content in Perches.

If your Map be laid down by a Scale of 35 Perches to an Inch, every Square Inch will contain 1225 Perches, and every ſmall Square 12.25 Perches; wherefore to 12 times the Number of ſmall Squares, add ¼ of their Number, and you have the Content in Perches.

If your Map be laid down by 40 Perches to an Inch, every Square Inch will contain 1600 Perches, and every ſmall Square 16 Perches; the Number of ſmall Squares therefore being multiplied by 16, gives the Content in Perches.

Or cut off the laſt Figure from the Sum of the ſmall Squares, and the other Figures are Acres; multiply that laſt Figure by 4, adding in 1, 2, or 3, for ¼, ½, or ¾, as before, and cut off 1 Figure to the Right from the Product, and the other, if any, are Roods: Multiply the laſt Figure by 4, and the Product is Perches.

For if any Number be divided by 10, which is done by cutting off the laſt Figure, it will quote the ſame as if that Number were multiplied by 16, and divided by 160.

Suppoſe the Number of ſmall Squares in a Map be 3994 ½, required the Content in Acres.

Acres 399|4 ½
4

Rood 1|8
4

Perches 32

After the like Manner you may find the Content of any Map, by knowing the Scale by which it was laid down; for the Square of the Scale gives the Content of a Square Inch in Perches, the hundredth Part of which will be the Content of a ſmall Square.

I am confident I ſhall be judged to be too prolix on this Head; but as this Method has ſtood the Teſt of many Ages, and is ſtill uſed by many, I have for the Benefit of thoſe of the ſloweſt Capacity, explained it as fully as I was able.

Third

Third Method to determine the Area of Ground.

To determine the Area of a Piece of Ground, Having the Map given, by reducing it to one Triangle equal thereto, and thence finding it's Content.

Plate VIII. Fig. 5.

Let A B C D E F G H be a Map of Ground, which you would reduce to one Triangle equal thereto.

Produce any Line of the Map, as AH, both Ways: Lay the Edge of a parallel Ruler from A to C, having B above it: Hold the other Side of the Ruler, or that next you faſt: open 'till the ſame Edge touches B, and by it, with a Protracting Pin, mark the Point *b* on the produced Line: Lay the Edge of the Ruler from *b* to D, having C above it: Hold the other Side faſt open, 'till the ſame Edge touches C, and by it mark the Point *c*, on the produced Line. A Line drawn from *c* to D will take in as much as it leaves out of the Map.

Again, lay the Edge of the Ruler from H to F, having G above it, keep the other Side faſt, open 'till the ſame Edge touches G, and by it mark the Point *g*, on the produced Line: Lay the Edge of the Ruler from *g* to E, having F above it, keep the other Side faſt, open, 'till the ſame Edge touches F, and by it mark the Point *f*, on the produced Line. Lay the Edge of the Ruler from *f* to D, having E above it, keep the other Side faſt, open 'till the ſame Edge touches E, and by it mark the Point *e*, on the produced Line. A Line drawn from D to *e*, will take in as much as it leaves out. Thus have you the Triangle *c* D *e*, equal to the irregular Polygon A B C D E F G H.

If when the Ruler's Edge be applied to the Points A and C, the Point B falls under the Ruler, hold that Side next the ſaid Points faſt, and draw back the other to any convenient Diſtance; then hold this laſt Side faſt, and draw back the former Edge to B, and by it mark *b*, on the produced Line; and thus a Parallel may be drawn to any Point under the Ruler, as well as if it were above it. It is beſt to keep the Point of your Protracting Pin in the laſt Point in the extended Line, 'till you lay the Edge of the Ruler from it to the next Station, or you may miſtake one Point for another.

This may alſo be performed with a Scale, or Ruler, which has a thin ſloped Edge, called a Fiducial, or ſure Edge; and a fine pointed Pair of Compaſſes. Thus,

Lay that Edge on the Points A and C, take the Diſtance from the Point B to the Edge of the Scale, ſo as it may only touch it, in the ſame Manner as you take the Perpendicular of a Triangle; carry that Diſtance down by the Edge of the Scale parallel to it, to *b*; and there deſcribe an Arc on the Point *b*, and if it juſt touches the Ruler's Edge, the Point *b* is in the true Place of the extended Line. Lay then the Fiducial Edge of the Scale from *b* to D, and take a Diſtance from C, that will juſt touch the Edge of the Scale; carry that Diſtance along the Edge, 'till the Point which was in C, cuts the produced Line in *c*; keep that Foot in *c*, and deſcribe an Arc, and if it juſt touches the Ruler's Edge, the Point *c* is in the true Place of the extended Line. Draw a Line from *c* to D, and it will take in and leave out equally: In like Manner the other Side of the Figure may be ballanced by the Line *e* D.

Let the Point of your Compaſſes be kept to the laſt Point on the extended Line, 'till you lay your Scale from it to the next Station, to prevent Miſtakes from the Number of Points.

That the Triangle *c* D *e*, is equal to the right-lined Figure A B C D E F G H, will be evident from Problems 18, 19. Sect. 1: For thereby, if a Line were drawn from *b* to C, it will give and take equally, and then the Figure *b* C D E F G H, will be equal to the Map. Thus the Figure is leſſened by one Side, and by the next Ballance Line will leſſen it by two, and ſo on, and will give and take equally. In the ſame Manner an Equality will ariſe on the other Side.

The Area of the Triangle is eaſily obtained, as before, and thus you have the Area of the Map.

It is beſt to extend one of the ſhorteſt Lines of the Polygon, becauſe if a very long Line be produced, the Triangle will have one Angle very obtuſe, and conſequently the other two very acute; in which Caſe it will not be eaſy to determine exactly the Length of the longeſt Side, or the Points where the Ballancing Lines cut the extended one.

This Method will be found very uſeful and ready in ſmall Encloſures, as well as very exact; it may be alſo uſed in large ones, but great Care muſt be taken of the Points on the extended Line, which will be crowded, as well as of not miſſing a Station.

Fourth Method to determine the Area of Ground.

To determine the Area of a Piece of Ground, having the Map thereof given, by a Scale of Acres.

In this Caſe let your Parallels be half an Inch aſunder, and (if it will admit it) let your Map be laid down by a Scale of 20 Perches to an Inch: Provide yourſelf with a tranſparent Horn, with Parallels on it at half an Inch aſunder, to which the Ends of the Horn muſt be at right Angles; an Inch Horn may be made to ſerve this Purpoſe. Between two of the middle Paraliels let a fine parallel Line be drawn, equi-diſtant to each, or a Quarter of an Inch from either. Apply the middle Parallels on the Horn between any two Parallels on the Map, and bring the End of the Horn ſo to the Lines of the Map between the ſaid Parallels, that you may have as much under it, as you eſtimate to be without it; and with a fine Pencil Point, or Protracting Pin, draw a Line between the two Parallels by the End of the Horn. You muſt be careful in keeping the Parallels on the Horn exactly over the Parallels on your Map. Do the like by the Lines of the Map between all the Parallels on it, till you have reduced what was between them into equivalent right-angled Parallelograms.

Plate VIII. Fig. 1.

Let ABCD repreſent a Map of Ground which is laid down by a Scale of 20 Perches to an Inch, and let the parallel Lines repreſented in the Scheme, be ſuppoſed to be half an Inch aſunder: By this Hypotheſis therefore it will be plain, that the Breadth between any two Parallels will be 10 Perches; and becauſe 160 Perches make an Acre, it follows, that 16 Perches in Length between any two Parallels will be an Acre.

Plate VIII. Fig. 3.

Make a Scale of equal Parts, whoſe Diviſions ſhall be 16 Perches, or 8 Tenths of an Inch aſunder; and ſubdivide one of thoſe Diviſions into Tenths, and half Tenths: The Parts between may be eſtimated near enough. Or one of the Diviſions may be divided diagonally into 100 equal Parts. Number the firſt large Diviſion 1, the ſecond 2, the third 3, *&c.* ſo every one of the large Diviſions will be an Acre, and the ſmall ones will be decimal Parts of an Acre.

Plate VIII. Fig. 2. repreſents the two middle Parallels on the Horn, with the intermediate Parallel at a Quarter of an Inch from each.

Apply this Part of the Horn between any two Parallels on the Map, Fig. I. 'till by the Edge thereof you find by Eſtimation you have as much at the Boundaries of the Map under it, as you leave out; and by the End of the Horn draw a Ballance Line with a Pencil, as before, and do the like between every two Parallels. The Pricked Lines in the Scheme repreſent the Ballance Lines.

Where the Boundaries are very crooked, as between *a* and *b*, ballance them with the Line *ab*, by the foregoing Method; bring the Edge of the middle Parallels on the Horn to this Line, 'till the intermediate Line meets *ab*, in *c*; then by the ſaid Edge draw the Line *de*, and the Part left out will be equal to that taken in; which muſt be manifeſt by what has already been ſaid. And this Method may be uſed wherever the Parts left out, and taken in, are too great to be eſtimated.

If an oblique Right Line be the Boundary between two Parallels, as the Line B *f* is, bring the Edge of the middle Parallels on the Horn to this Line, 'till the intermediate Line meets it; and by the Edge draw the Ballance Line.

Having reduced what lies between every two Parallels to an Oblong equal thereto, number them with 1, 2, 3, 4, *&c.*

Then either apply the Fiducial Edge of your Scale where the Diviſions of Acres are, to find the Length of each Oblong; or meaſure the Lengths from your Scale of Acres, which you have divided diagonally: Set theſe ſeverally oppoſite to their reſpective Numbers, and their Sum will be the Acres and decimal Parts of an Acre contained in all the Oblongs. Thus,

No. 1.

No.	
1	.82
2	2.23
3	3.15
4	4.08
5	3.95
6	1.92
7	.92
	17.07
	4
	.28
	.40
	11.20

A. R. P.
Content 17. 0. 11.

If there be a ſmall Piece above or below, that won't reach a Parallel, or if there be any long irregular Excreſcences on either Side, let theſe be caſt up by Triangles, and their Content be added to the Sum of the Oblongs, for the Content of the Map.

If your Map be laid down by a Scale of forty Perches to an Inch, it is beſt to have Quarter Inch Parallels, and then every Acre will be 4 Tenths of an Inch in Length, which may be laid on the other Edge of the Scale.

Let your Map be laid down by what Scale ſoever, if you caſt it up by the large Scale of Acres, or by that of 20 Perches to an Inch, the true Content may be eaſily obtained, as will hereafter be ſhewn.

The larger your Scale is by which your Map is laid down, the more certain the Content will be by any Method whatſoever.

When a Perſon is ſome Time accuſtomed to this Method, he will prefer it to any of the foregoing for it's Expedition and Certainty.

If you be unprovided with a Scale of Acres, you may meaſure the Lengths of the Oblongs by the Diviſions on the Edge of the Protractor; and if the Map be ruled with half Inch Parallels, the Sum of the Lengths of the Oblongs being divided by 16, will give the Content in Acres, if the Map was laid down by 20 Perches to an Inch; but if by 40, divide the Sum by 4, and you have the Content required.

Fifth Method to determine the Area of Ground.

To determine the Area of a Piece of Ground, having the Map given, by Weight.

Let the Map, Plate VIII. Fig. 6. be that whoſe Area is required.

Let Parallels be drawn at half an Inch aſunder, and others at Right-Angles to thoſe at a like Diſtance; ſo will each Square be a Quarter of a Square Inch in Area.

With a Pair of fine Sciſſars, or a Penknife, cut away all the Squares marked *a*, which the Bounds of the Map do not reach, as unneceſſary and uſeleſs; then will remain all the whole Squares contained within the Body of the Map, and the Squares which the Bounds paſs through, which are mark'd z, z, z, *&c.*

Count the Number of whole Squares in the Body of the Map, and thoſe marked z, together, which Number Note down; then find with Grains and Tenths of a Grain, the Weight of all that Paper. Call the Number of Squares, firſt Area, and their Weight in Grains and Parts, call their firſt Weight.

Cut the Map then cloſe by the Bounds, ſo will the Pieces of Squares without the Bounds, marked z, be cut away, and the Map only remain, which weigh in Grains and Decimals of a Grain, as before. Then ſay, as the firſt Weight is to the firſt Area, ſo is the ſecond Weight to the ſecond Area, which gives the Area of the Map in Squares, and Decimals of a Square. Then by knowing by what Scale it was laid down, the Area of each Square is known, and conſequently the Area of the whole Map. The Area of each Square by the different Scales moſt uſed, will be thus:

By a Scale of	Perch to an Inch, every Square will be	
8	16	Perches.
10	25	
16	64	
20	100	
40	400	
80	10	Acres.

The following was an Experiment made a few Years ſince upon the Map, Plate X. Fig. 3, which when caſt up by the following univerſal Methods, gives 110 A. 2 R. 32 P. The Map was laid down by a Scale of 20 Perches to an Inch.

Number of Squares in the Map, and thoſe marked z, 223.

	Grains.
Weight of the Paper	74.6
Weight of the Map	59.2

Then 74.6 : 223 : : 59.2 : 176.96 Squares in the Map, and each Square being 100 Perches, makes 17696 Perches, or 110 A. 2 R. 16 P. differing only 16 Perches from the true Content.

This may alſo be performed by drawing a Trapezium about the Map, and finding it's Content as before, cutting it out and weighing it, and afterwards the Map; by ſaying, as the Weight of the Trapezium is to the Content of the Trapezium, ſo is the Weight of the Map to the Content of the Map. But this Method will not be ſo exact as the foregoing one; for there will be a much greater Space, or Area comprehended between the Trapezium and the Map, than that contained in the Parts of Squares thro' which the Lines of the Map run, and the Map; therefore, as there are ſome Inequalities in all Paper, the greater the Error will be.

Tho' this Method of Weighing may ſeem whimſical and ridiculous, yet if Experiments be made with nice Scales and Weights, upon Maps laid down by large Scales, and drawn upon good even *Writing-Demy* Paper, the Contents produced will be found to be much nearer the Truth than can be imagined.

SECT. V.

Containing Four new conciſe Methods of determining the Areas of Right Lined Figures univerſally, or by Calculation: With ſome neceſſary previous Definitions and Theorems.

CALCULATION.

DEFINITIONS.

Plate VIII. Fig. 7.

I. MERIDIANS are North and South Lines, which are ſuppoſed to paſs through every Station, running parallel to each other.

II. The Difference of Latitude, or the Northing or Southing of any ſtationary Line, is the Diſtance that one End of the Line is North or South from the other End; or it is the Diſtance which is intercepted on the Meridian, between the Beginning of the ſtationary Line, and a Perpendicular drawn from the other End to that Meridian. Thus, if NS be a Meridian Line paſſing through the Point A of the Line AB, then is A*b* the Difference of Latitude, or Southing of that Line.

III. The

III. The Departure of any ſtationary Line, is the neareſt Diſtance from one End of the Line, to a Meridian paſſing through the other End. Thus, B*b* is the Departure, or Eaſting of the Line AB.

But if CB be a Meridian, and the Meaſure of the ſtationary Diſtance be taken from B to A; then is BC the Difference of Latitude, or Northing, and AC the Departure, or Weſting of the Line BA.

IV. That Meridian which paſſes through the firſt Station, is ſometimes called the firſt Meridian; and ſometimes it is a Meridian paſſing on the Eaſt or Weſt Side of the Map, at the Diſtance of the Breadth thereof, from Eaſt to Weſt, ſet off from the firſt Station.

V. The Meridian Diſtance of any Station is the Diſtance thereof from the firſt Meridian, whether it be ſuppoſed to paſs through the firſt Station, or on the Eaſt or Weſt Side of the Map.

THEO. I.

In every Survey which is truly taken, the Sum of the Northings will be equal to that of the Southings; and the Sum of the Eaſtings equal to that of the Weſtings.

Plate IX. Fig. 1.

Let *a b c e f g* repreſent a Plot, or Parcel of Land. Let *a* be the firſt Station, *b* the ſecond, *c* the third, *&c.* Let NS be a Meridian Line, then will all

Lines parallel thereto, which paſs through the ſeveral Stations, be Meridians alſo; as ao, bs, cd, &c. and the Lines bo, cs, de, &c. perpendicular to thoſe, will be Eaſt or Weſt Lines, or Departures.

The Northings $ei + go + bq = ao + bs + cd + fr$ the Southings. Thus:

Let the Figure be compleated; then it is plain that $go + bq + rk = ao + bs + cd$, and $ei - rk = fr$. If to the former Part of this firſt Equation $ei - rk$ be added, and fr to the latter, then $go + bq + ei = ao + bs + cd + fr$; that is, the Sum of the Northings is equal to that of the Southings.

The Eaſtings $cs + qa = ob + de + if + rg + ob$, the Weſtings. Thus:

For $aq + yo\ (az) = de + if + rg + ob$, and $bo = cs - yo$. If to the former Part of this firſt Equation $cs - yo$ be added, and bo to the latter, then $cs + aq = ob + de + if + rg + ob$; that is, the Sum of the Eaſtings is equal to that of the Weſtings. Q. E. D.

SCHOLIUM.

This Theorem is of Uſe to prove, whether the Field-Work be truly taken, or not: For if the Sum of the Northings be equal to that of the Southings, and the Sum of the Eaſtings be equal to that of the Weſtings, the Field-Work is truly taken, otherwiſe not.

Since the Proof and Certainty of a Field-Work depends on this Truth, it will be neceſſary to ſhew how the Difference of Latitude, and Departure, for any ſtationary Line, whoſe Bearing and Diſtance are given, may be obtained by Trigonometry, or by Tables hereunto annexed.

PROBLEM.

To find the Difference of Latitude and Departure, the Bearing and ſtationary Diſtance being given, by Trigonometry.

Plate IX. Fig. 1.

Let the Bearing of the Diſtance Line *ab*, be SW 16° ¾, and it's Length in Four-Pole Chains, 10 C. 12 L. Required *ao*, the Difference of Latitude, and *ob*, the Departure.

By Caſe 1. of *Rectangular Trigonometry*.

R	:	*ab*	: :	S. *oab*	:	*bo*	
90°		10.12		$16^{\circ}.45^{l}$		2.92	Dep.
R.	:	*ab*	: :	S. *abo*	:	*ao*	
90°		10.12		$73^{\circ}.15^{l}$		9.69	Diff. Lat.

Thus the Difference of Latitude, or Southing, is found to be 9 C. 69 L. and the Departure, or Weſting, 2 C. 92 L. And in like Manner the Difference of Latitude and Departure may be found for any ſtationary Diſtance, having the Bearing, and Length thereof given.

This may alſo be performed by *Gunter*'s *Scale*, as already ſhewn, in Caſe 1. of *Rectangular Trigonometry*.

2. *To*

2. *To find the Difference of Latitude and Departure, by Help of the annexed Table.*

In the firſt Column find the Degree, and the ¼, ½, or ¾ of a Degree contained in the Bearing; in a Line with which, under N. S. and E. W. you have the Difference of Latitude, and half the Departure for that Bearing, the Diſtance being 1 Four-Pole Chain.

Therefore, if the correſponding Numbers, oppoſite to the Degrees in the Table, be ſeverally multiplied by the Length of the ſtationary Diſtance, their reſpective Products will be the Difference of Latitude, and Half Departure, for that Line. Thus,

Let the Bearing as before be SW. 16° ¾, Diſtance 10 C. 12 L. Required the Difference of Latitude and Departure.

Oppoſite to 16° ¾, are theſe Numbers.

.9576 N. S.	.1441 E. W.
10.12	10.12
19152	2882
9576	1441
9576	1441
9.690912 Southing	1.458292 Half Weſting.
	2.916584 Weſting.

THEO.

THEO. II.

If a Map lies to the Eaſt of any firſt Meridian, the farther any Point be taken in the ſeveral ſtationary Diſtances that have Eaſt Departure, the farther will that Point be from the firſt Meridian; but the farther any Point be taken in ſtationary Diſtances that have Weſt Departure, the nearer that Point will be to the firſt Meridian.

Plate IX. Fig. 2.

Let the firſt Meridian be N. S, and let the Stations *a*, *b*, *c*, lie on the Eaſt Side thereof; let the Perpendiculars *af*, *bm*, *cn*, and *dk* be drawn; draw *ao*, *bi*, and *cp* parallel to NS.

The Departure of the firſt Line *ab* is *ob* and lies to the Weſt Side of it's Meridian *ao*, which paſſes through the Station *a*: But the Point *b*, or any Point in the Line *ab*, is nearer to the firſt Meridian NS, than the Point *a* is; for $af\ (om) - ob = bm$, therefore *bm* is leſs than *af*, and conſequently the Point *b* is nearer N. S, than the Point *a* is.

The Departure of the next Station is *ci*, which is to the Eaſtward of it's Meridian *bi*, which paſſes through the Station *b*; but the Point *e*, or any Point in the Line *bc*, is more remote from the firſt Meridian NS, than the Point *b* is; for $bm + ci = cn$.

Therefore when the Departure of any ſtationary Diſtance is Eaſt, the further you proceed on in the Diſtance Line, the more you remove from the firſt Meridian. But when the Departure is Weſt, the farther you proceed on in the Diſtance Line, the nearer you approach the firſt Meridian. Q. E. D.

COR. I.

Hence if the Diſtance of the firſt Station from the firſt Meridian be known, and alſo the ſeveral Departures ob, ci, pd; the Diſtances of the Points b, c, and d, from that Meridian may be likewiſe known: For the firſt Departure being Weſt, $af - bo = bm$; the next Departure ie being Eaſt, $bm + ci = cn$; and the next Departure being Weſt, $cn - pd = dk$. And if you proceed in the like Manner to add the Eaſt, and ſubtract the Weſt Departure in any Survey, and the Diſtance of any Station from the firſt Meridian may be known.

COR. II.

As by the whole Departure of each ſtationary Diſtance, with the Diſtance of any one Station from the firſt Meridian, the Diſtance of every Station in the Map from the firſt Meridian may be known; ſo by having Half the Departure of each ſtationary Line given, with the Diſtance of the firſt (or any other) Station from the firſt Meridian, the Diſtance of the Middle of each ſtationary Line, from the firſt Meridian may be found thus. If bq be Half the Departure of the firſt Station, the Point b will be in the Middle between a and b; for $ob : ba :: bq : ba$, but ob being double of bq, ba will be double of ba.

Again, $ob : oa :: bq : qa$, wherefore oa will be double of qa, and qg will fall in the Middle between af and bm: And for the ſame Reaſon, admitting lz to be Half the Departure of the ſecond ſtationary Line; le will fall in the Middle between bm and cn; and conſequently if Half the Departure of each Station be given, the Lines gb, mb, el, nc, &c. will alſo be given, for $af - qb = bg$. $bg - bq\ (kh) = bm$. $bm + lz = le$. $le + lz\ (xc) = cn$. $cn - ry = rs$. $rs - ry = dk$.

LEMMA.

In every Trapezium ABCD, having two Sides, AD, BC, perpendicular to the ſame Side AB, Half the Sum of thoſe perpendicular Sides, viz. IF, multiplied into AB, the Baſe on which they ſtand, will be the Area of the Trapezium ABCD.

Plate VIII. Fig. 10.

Let DK be drawn parallel to AB, and let it be biſected by the Perpendicular FHI; it is plain, from ſimilar Triangles, that it will alſo biſect DC in F: Through F, draw FG parallel to DK, and this will alſo biſect CK in G.

It is manifeſt that KC is equal to the Difference of the Sides BC, AD, and GK = GC = ED their Half Difference: Therefore (by Cor. to Theo. 4. Sect. 2.) CB—CG = GB = FI, will be half the Sum of AD and CB; and it has been ſhewn that DFE, GFC are equilateral, and hence they are equal to each other. But AB multiplied by FI, or BG, Half the Sum of the Sides AD, and BC, will be the Area of the Rectangle AEGB = ABCD: For if from the Trapezium ABCD, the Triangle FGC be taken, and in lieu thereof DEF be added, the Rectangle AEGB, which is made of Half the Sum of the Sides into the Baſe, will be equal to the Trapezium ABCD. Q. E. D.

THEO. III.

If the Meridian Diſtance taken in the Middle of every ſtationary Line, be multiplied into the particular Northing or Southing of that Line; and if the Difference between the Sum of the North and the Sum of the South Products be taken, it will be the Area of the Survey.

Plate IX. Fig. 3.

Let ABCDEFGA be the Map of a Survey whoſe Area is required.

To the firſt Meridian NS, drawn on the Weſt Side of the Map, let Perpendiculars be drawn from the Beginning and Middle of every ſtationary Line; let alſo Meridians be drawn through each Station, and the Figure be compleated.

The Northings are A*b* = WM, B*d* = MN, F*q* = QS, and G*g* = SW.

The Meridian Diſtances to the Middle of every ſtationary Line tending to the North, are IY, KO, P*a* and HV.

The Northings (F*q*) QS multiplied by P*a* = Area of FGSQF, (by the Lemma); and (G*g*) SW multiplied by HV = Area SGAWS. (A*b*) WM multiplied by IY = Area WABMW, and (B*d*) MN multiplied by KO = Area MECNM; the Sum of all theſe Areas is the whole North Area QFGABCNQ.

In the ſame Manner the Southings (C*b*) NZ multiplied by LL = Area NCDZN, (D*m*) ZT multiplied by MX = the Area ZDETZ, (E*n*) TQ multiplied by RR = the Area TEFQT, the Sum of all theſe Areas is the South Area NCDEFQN.

The Area of NCDEFQN = QFGABCNQ = Area of the Map ABCDEFA. Q. E. D.

The Demonſtration of this Theorem in a more complicated Figure.

Plate IX. Fig. 4.

Let *a* be the firſt Station, *b* the ſecond, *c* the third, *&c.* and the Line NS the firſt Meridian.

The Figure made of the ſeveral Southings, into the Meridian Diſtances to the Middle of every ſtationary Line is *ocdefgho*+*keftk*. And the Figure made of the ſeveral Northings into the Meridian Diſtance to the Middle of every ſtationary Line, is *bgabcob*+*keftk*.

The Difference of theſe Areas is *ocdefgho* — *bgabcob* = *abcdefga*, the Area of the Map. Q. E. D.

If the firſt Meridian *ob* had paſſed either through the extreme Weſt Point *a*, or the Eaſtermoſt Point *g*; the Demonſtration would be the ſame, both in this and the laſt Caſe.

SCHOLIUM.

From theſe Theorems we learn how to find the Area of a Piece of Ground by Calculation or without a Map, when the firſt Meridian paſſes on the Eaſt or Weſt Side of a Map.

To find the Area of Ground from the Field-Notes, by Calculation, or without the Aſſiſtance of a Map.

In the following Field-Book, Method No. 1.

Transfer the Letters of the Bearing which are in the ſecond Column, from thence to the fourth, and place the firſt Letter thereof over the ſecond: Thus, the firſt Bearing being NE, in the fourth Column, or that of Latitude and Departure, write down the Letter N, and under it E; the firſt ſhews that the Difference of Latitude is Northing, and the ſecond that the Departure is Eaſting: The like is done with the ſecond Bearing, being NE alſo. The third Bearing being Eaſt, has neither Northing or Southing; therefore in the Place of either ſet down Cyphers only. The fourth Bearing being SW, write down S over W; and ſo proceed to ſet down the initial Letters of every Bearing one over the other: But if a Bearing be N, S, E, or W, put down ſuch a Letter, and Cyphers in the Place of the ſecond Letter.

Before or after every Letter E, place the Sign +; and alſo before or after every W, place the Sign—; to ſhew that every Eaſting is to be added, and every Weſting ſubſtracted.

By Trigonometrical Calculation, or from the annexed Tables, find the difference of Latitude and Half Departure of each Line, and place them after their reſpective Letters. Thus the firſt Bearing being N. E. 75, Diſtance 13 C. 70 L. the Northing will be 3 C. 54 L. and Half the Eaſting 6 C. 6 L. the former of which place after the Letter N, and the latter after E: and ſo proceed with all the reſt, 'till the Column of Difference of Latitude and Departure be compleat.

N. B. It has been already ſhewn, that the Numbers under the Column EW, in the annexed Table, are half the Eaſtings or Weſtings, or Half the Departure; but for Brevity ſake, we ſhall hereafter call them Eaſtings or Weſtings, and ſometimes Departures.

On a looſe Piece of Paper write down the Letters N, S, E, and W, at any convenient Diſtance aſunder: and from the Column of Latitude and Departure collect all the Northings, Southings, Eaſtings, and Weſtings; add them up ſeverally, and if the Northings are equal to the Southings, and the Eaſtings to the Weſtings, the Work is truly taken, otherwiſe not, as before demonſtrated.

The Latitudes and Departures in the following Field-Book, are thus collected.

N.	S.	E.	W.
3.54	29.44	6.61	9.74
9.65	3.87	1.80	7.76
9.00	1.21	8.10	5.77
6.94	2.75	5.74	4.64
15.38	10.48	2.68	1.40
12.93	9.69	5.84	1.46
57.44	57.44	30.77	30.77

To find the Numbers for the Column entitled Meridian Diſtance.

Oppoſite Number 0 in your Table, place 61 C. 54 L. the whole Departure, or double the Sum of the Half Eaſtings, or Half Weſtings; in the Column of Meridian Diſtance.

In Plate IX. Fig. 3. Let AW repreſent this firſt Number, *viz.* 61 C. 54 L. and NQ. the firſt Meridian Line; and ſince the Map lies to the Eaſt Side of that Meridian, (by Theo. 3. of this Section) thoſe Lines that have Eaſt Departure will lie farther from the firſt Meridian, than thoſe that have Weſt Departure; and therefore knowing the Length of the Line AW, the Lengths of the other Lines IY, BM, *&c.* may be found, by adding the Eaſtings, and ſubſtracting the Weſtings, by Cor. 2. Theo. 2. of this Section.

The firſt Meridian is ſuppoſed to be the Length of the whole Departure, or the entire Eaſting or Weſting, from the firſt Station: For ſhould the firſt Station be at the Eaſtermoſt Point of the Land, the firſt Meridian will then paſs thro' the Weſtermoſt, and the Map will be entirely on the Eaſt of the firſt Meridian: But were the Meridian Diſtance leſs than whole Eaſting or Weſting, and the firſt Station at the Eaſtermoſt Point of the Land, then it is plain, that the firſt Meridian will paſs through the Map, and Part thereof will lie on the Eaſt, and the reſt on the Weſt of the ſaid firſt Meridian. In this Caſe therefore no Number leſs than the whole Eaſting or Weſting will be ſufficient for the Meridian Diſtance, in order that the Map ſhould lie entirely on the Eaſt thereof. But if the firſt Station be not the Eaſtermoſt Point of the Land, a leſs Number might be ſufficient for the Meridian Diſtance, but how much leſs would be troubleſome and needleſs to determine; ſince the Meridian Diſtance being made equal to the entire Eaſting or Weſting will anſwer in all Caſes, let the firſt Station be where it will; as it leaves the Map entirely to the Eaſt of the firſt Meridian.

The ſame Reaſoning will hold good, if you would have the Map to lie entirely on the Weſt of the firſt Meridian.

In the following Table or Field-Book the Eaſting of the firſt ſtationary Line is marked+6.61, muſt be added to 6..54, and placed in a Line with N 3.54, and 68.15+6.61=74.76 muſt be placed in a Line with E+6.61.

Again, 74.76+1.80=76.56, which place in a Line with 9.65; and 76.56+1.80=78.36, which place in a Line with E+1.80. Proceed in the like Manner to add the Eaſtings, and ſubſtract the Weſtings,

ings, 'till you have compleated the Column of Meridian Diſtance; the laſt Number of which will be equal to the firſt, if the Work be right, becauſe there was juſt as much added, as ſubſtracted, ſince the Sum of the Eaſtings is equal to that of the Weſtings.

The Numbers in the Column of Meridian Diſtance which are in the ſame Line with the Northings or Southings, in the Column of Latitude and Half Departure, are the Meridian Diſtances in the Middle of each Line; and thoſe in the ſame Column which are in a Line with the Eaſtings or Weſtings, are the Meridian Diſtances at every Station, or at the Beginning of every Line: And ſince it has been demonſtrated in Theo. 3. of this Sect. that if the Meridian Diſtance taken in the Middle of every Line, be multiplied into the particular Northing or Southing of that Line, that the Difference of the Sums of the North and South Products, will be the Area of the Map; it follows, that the upper Number of every two which ſtand in the Column of Meridian Diſtance, muſt be multiplied by the Northing or Southing adjoining it. Theſe ſeveral Products muſt be put in the Columns of N. Area, or S. Area, according as the Difference of Latitude is North or South. Thus if 68.15, the Meridian Diſtance to the Middle of the firſt Line, be multiplied by 3.54, the Northing adjoining it, the Product 241.2510, muſt be put in the Column of North Area; but in the 4th Stationary Line, when 84.82 is multiplied by 29.44, the Southing of that Line, the Product 2497.1008 muſt be placed in the Column of S. Area, and ſo of the reſt.

Laſtly, if you add up the Columns of N. and S. Area; the Difference of theſe will be the Area of the Survey in ſquare Four-Pole Chains, which may be eaſily reduced into Acres as before.

Thus the Content of the following Field-Book is 1107.05 13 ſquare Four-Pole Chains, or 110.70513 Acres.

```
Acres   110.70513
                4
        ---------
Roods     2.82052
                4
        ---------
Perches  32.8208
        ---------
```

A. R. P.
Or 110. 2. 32. or nearly 33 Perches.

No. Sta.	Bearings	C. L.	Latit. and Half Dep.	Mer. Dist.	N Area.	S Area.
0				61.54		
1	NE 75	13.70	N 3.54 E+ 6.61	68.15 74.76	241.2510	
2	NE 20½	10.30	N 9.65 E+ 1.80	76.56 78.36	738.8040	
3	East	16.20	0.00 E + 8.10	86.46 94.56		
4	SW 33½	35.30	S 29.44 W— 9.74	84.82 75.08		2497.1008
5	SW 76	16.00	S 3.87 W— 7.76	67.32 59.56		260.5284
6	North	9.00	N 9.00 0.00	59.56 59.56	536.0400	
7	SW 84	11.60	S. 1.21 W— 5.77	53.79 48.02		65.0859
8	NW 53¼	11.60	N 6.94 W— 4.64	43.38 38.74	301.0572	
9	NE 36¾	19.20	N 15.38 E+ 5.74	44.48 50.22	684.1024	
10	NE 22½	14.00	N 12.93 E+ 2.68	52.90 55.58	683.9970	
11	SE 76¾	12.00	S 2.75 E+ 5.84	61.42 67.26		168.9050
12	SW 15	10.85	S 10.48 W— 1.40	65.86 64.46		690.2128
13	SW 16¾	10.12	S. 9.69 W— 1.46	63.00 61.54		610.4700
					3185.2516	4292.3029
					Subt	3185.2516
					Area in square Four-Pole Chains	1107.0513
					Area in Acres and Decimal Parts	110.70513

The foregoing FIELD-BOOK, *by a ſecond Method.*

The Content of the foregoing Field-Book may as well be found by putting + to the Weſtings, and — to the Eaſtings; that is, by adding the Weſtings and ſubtracting the Eaſtings, which is plain, if you conceive the firſt Meridian to be on the Eaſt Side of the Map. Thus,

No. Sta.	Bearings	C. L.	Latit. and Half Dep.	Mer. Dift.	N. Area.	S. Area.
0				61.54		
1	NE 75	13.70	N 3.54 E — 6.61	54.93 48.32	194.4522	
2	NE 20¾	10.30	N 9.65 E — 1.80	46.52 44.72	448.9180	
3	East	16.20	0.00 E — 8.10	36.62 28.52		
4	SW 33½	35.30	S 29.44 W + 9.74	38.26 48.00		1126.3744
5	SW 76	16.00	S 3.87 W + 7.76	55.76 63.52		215.7912
6	North	9.00	N 9.00 0.00	63.52 63.52	571.6800	
7	SW 84	11.60	S 1.21 W + 5.77	69.29 75.06		83.8409
8	NW 53¼	11.60	N 6.94 W + 4.64	79.70 84.34	553.1180	
9	NE 36¾	19.20	N 15.38 E — 5.74	78.60 72.86	1208.8680	
10	NE 22½	14.00	N 12.93 E — 2.68	70.18 67.50	907.4274	
11	SE 76¼	12.00	S 2.75 E — 5.84	61.66 55.82		169.5650
12	SW 15	10.85	S 10.48 W + 1.40	57.22 58.62		599.6656
13	SW 16¾	10.12	S 9 69 W + 1.46	60.08 61.54		582.1752
					3884.4636	2777.4123
				Subtract	2777.4123	
Content as before in Four-Pole Chains					1107.0513	
				Content in Acres	110.70513	

Remarks on the two foregoing Methods.

1. That the Sum of the South Area Column in the firſt, and North Area in the ſecond, are the greateſt.

2. The Sum of the North Area Column in the firſt, added to the Sum of the North Area Column in the ſecond, is equal to the Sum of the South Area Column in the firſt, added to the Sum of the South Area Column in the ſecond. Thus,

1.	N. Area Column	3185.2516
2.	N. Area Column	3884.4636
	Sum	7069.7152

1.	S. Area Column	4292.3029
2.	S. Area Column	2777.4123
	Sum	7069.7152

3. That if the Sum of the North Area Column of the firſt, be added to the Sum of the South Area of the ſecond, and this Sum be taken from a Product made by the Multiplication of twice the whole Meridian Diſtance, or twice the entire Eaſting or Weſting, into the Sum of the whole Northing or Southing, the Remainder will be the Area of the Survey. Thus,

Firſt Meridian Diſtance, or the entire Eaſting or Weſting, is 61.54, the Double of which is 123.

123 08. The Sum of the whole Northing or Southing is 57.44. Then,

123.08 multiplied by 57.44 = 7069.7152
3185.2516 + 2777.4123 = 5962.6639

Area of the Survey as before 1107.0513

4. If the Sum in the South Area Column in the firſt, be added to the Sum of the North Area Column in the ſecond, and from that Sum be ſubſtracted the former Product 7069.7152, the Remainder will be the Area of the Survey.

Sum of the South Area of the firſt 4292.3029
Sum of the North Area of the ſecond 3884.4636

The Sum 8176.7665

The Sum 8176.7665
The Product before found 7069.7152

Area as before 1107.0513

THEO IV.

When the firſt Meridian paſſes thro' the Map.

If the Eaſt Meridian Diſtances in the middle of each Line be multiplied into the particular Southing, and the Weſt Meridian Diſtances into the particular Northing, the Sum of theſe Products will be the Area of the Map.

Plate X. Fig. 1.

Let the Figure *abkm* be a Map, the Lines *ab*, *bk* to the Southward, and *km*, *ma* to the Northward, NAS the firſt Meridian Line paſſing through the firſt Station *a*.

$$\left.\begin{array}{r}\text{The Meridian}\\ \text{Diſtances Eaſt}\end{array}\right\}\left.\begin{array}{l} zd \times ao \\ tu \times ox(by)\end{array}\right\} = \text{Area}\left\{\begin{array}{l} am \\ aw\end{array}\right.$$

$$\left.\begin{array}{r}\text{The Meridian}\\ \text{Diſtances Weſt}\end{array}\right\}\left.\begin{array}{l} ef \times gx \\ hh \times ga(my)\end{array}\right\} = \text{Area}\left\{\begin{array}{l} xp \\ gl\end{array}\right.$$

Theſe four Areas $am+ow+xp+gl$ will be the Area of the whole Figure *cmſwiprlc*, which is equal to the Area of the Map *abkm*. Compleat the Figure.

The Parallelograms *am* and *ow*, are made of the Eaſt Meridian Diſtances *dz* and *tu*, multiplied into the Southings *ao* and *ox*. The Parallelograms *xp* and *gl* are compoſed of the Weſt Meridian Diſtances *ef* and *bb*, multiplied into the Northings *xg* and *ga (my)* but theſe four Parallelograms are equal to the Area of the Map; for if from them be taken the four Triangles marked Z, and in the Place of thoſe be ſubſtituted the four Triangles marked O, which are equal to the former; then it is plain the Area of the Map will be equal to the four Parallelograms. Q. E. D.

THEO. V.

If the Meridian Diſtance when Eaſt, be multiplied into the Southings, and the Meridian Diſtance when Weſt, be multiplied into the Northings, the Sum of theſe leſs by the Meridian Diſtance when Weſt, multiplied into the Southings, is the Area of the Survey.

Plate X. Fig 2.

Let *a b c* be the Map.

The Figure being compleated, the Rectangle *af* is made of the Meridian Diſtance *eq*, when Eaſt multiplied into the Southing *an*; the Rectangle *yk* is made of the Meridian Diſtance *xw*, multiplied into the Northing *cz* or *ya*. Theſe two Rectangles, or Parallelograms, *af+yk*, make the Area of the Figure *dfnyikd*, from which taking the Rectangle *oy*, made of the Meridian Diſtance *tu*, when Weſt into the Southing *ob* or *bm*, the Remainder is the Area of the Figure *dfobikd*, which is equal to the Area of the Map.

Let *bou* = Y, *urib* = L, *ric* = C, *wrc* = Z, *akw* = K and *efb* = B, *ade* = A. I ſay, that Y+Z+B = K + L + A.

Y = L + O, add Z to both, then Y + Z = L + O + Z; but Z + O = K, put K inſtead of Z + O, then Y + Z = L + K, add to both Sides the equal Triangles B and A, then Y + Z + B = L + K + A. If therefore B + Y + Z be taken from *abc*, and in lieu thereof we put L + K + A, we ſhall have the Figure *dſobikd* = *abc*, but that Figure is made up of the Meridian Diſtance when Eaſt, multiplied into the Southing, and the Meridian Diſtance, when Weſt, multiplied into the Northing, leſs by the Meridian Diſtance, when Weſt, multiplied into the Southing. Q. E. D.

COROLLARY.

Since the Meridian Diſtance (when Weſt) multiplied into the Southing, is to be ſubſtracted, by the ſame Reaſoning the Meridian Diſtance when Eaſt, multiplied into the Northing, muſt be alſo ſubſtracted.

SCHOLIUM.

From the two preceeding Theorems we learn how to find the Area of a Map, when the firſt Meridian paſſes through it; that is, when one Part of the Map lies on the Eaſt and the other on the Weſt Side of that Meridian. Thus.

RULE.

The Merid. } Eaſt { multiplied { Southings }
Diſt. when } Weſt { into the { Northings }
their Sum is the Area of the Map.

But,

The Merid. { Eaſt } multiplied { Northings }
Diſt. when { Weſt } into the { Southings }
the Sum of theſe Products taken from the former, gives the Area of the Map.

Theſe Theorems are true, when the Surveyor keeps the Land he ſurveys on his Right Hand, which we ſuppoſe thro' the whole to be done; but if he goes the contrary Way, call the Southings Northings, and the Northings Southings, and the ſame Rule will hold good.

From what has been demonſtrated in Theo. 2. of this Section, it is plain, that if a Map lies to the Eaſt of the firſt Meridian, the Meridian Diſtances encreaſe when the Departures are Eaſt, and decreaſe when Weſt. And again, if the Map lies to the Weſt of the firſt Meridian, the Meridian Diſtances increaſe with Weſt Departure, and decreaſe with Eaſt; from whence this general Rule for Meridian Diſtances may be inferred.

1. The Meridian Diſtance and Departure, both Eaſt, or both Weſt, their Sum is the Meridian Diſtance of the ſame Name.

2. The Meridian Diſtance and Departure of different Names, that is, one Eaſt and the other Weſt, their Difference is the Meridian Diſtance, of the ſame Name with the greater.

Thus in the third Method of finding the Area, as in the following Field-Book.

The firſt Departure is put oppoſite to the Northing or Southing of the firſt Station, and is the firſt Meridian Diſtance of the ſame Name. Thus if the firſt Departure be Eaſt, the firſt Meridian Diſtance will be the ſame as the Departure, and Eaſt alſo; and if Weſt, it will be the ſame Way.

The firſt Meridian Diſtance	6 61	E
The next Departure	6.61	E
The ſecond Meridian Diſtance	13.22	E
The next Departure	1.80	E
The third Meridian Diſtance	15.02	E

At Station 5, the Meridian Diſtance	5.78	E
The next Departure	7.76	W
The next Meridian Diſtance	1.98	W

At Station 11, the Meridian Diſtance	0.12	W
The next Departure	5.84	E
The next Meridian Diſtance	5.72	E

Plate X. Fig. 3.

In the 5th and 11th Stations the Meridian Diſtances being leſs than the Departures, and of a contrary Name, the Map will croſs the firſt Meridian, and will paſs as in the 5th Line, from the Eaſt to the Weſt Side of the Meridian; and in the 11th Line it will again croſs from the Eaſt to the Weſt Side, which will evidently appear, if the Field-Work be protracted, and the Meridian Line paſſing through the firſt Station, be drawn through the Map.

The Field-Book caſt up by the Third Method, will be evident from the two foregoing Theorems, and therefore requires no farther Explanation; but *to find the Area by the* 4th *Method*, take this

RULE.

When the Meridian Diſtances are Eaſt, put the Products of North and South Areas in their proper Columns; but when Weſt, in their contrary Columns; that is, in the Column of South Area, when the Difference of Latitude is North; and in North Area when South: The Reaſon of which is plain, from the two laſt Theorems. The Difference of theſe two Columns will be the Area of the Map.

By comparing the Area of the Field-Book by the two firſt and the two laſt Methods, it is plain, that the Column of Deductions of the two laſt, are much leſs than either of the two former, and conſequently the Multiplications eaſier, as they conſiſt of leſs Figures. Sometimes it will happen that there are no Deductions, but all the Area will ſtand in one Column, which cannot happen in either of the two former Methods.

No. Sta.	Bearings	C. L.	Latit. and Half Dep.	Mer. Diſt.	Area.	Deductions
1	NE 75	13.70	N 3.54 E 6.61	6.61 E 13.22 E		23.3994
2	NE 20½	10.30	N 9.65 E 1.80	15.02 E 16.82 E		144.9430
3	Eaſt	16.20	0.00 E 8.10	24.92 E 33.02 E		
4	SW 33½	35.30	S 29.44 W 9.74	23.29 E 13.54 E	685.3632	
5	SW 76	16.00	S 3.87 W 7.76	5.78 E 1.98W	22.3686	
6	North	9.00	N 9.00 0.00	1.98W 1.98W	17.8200	
7	SW 84	11.60	S 1.21 W 5.77	7.75W 13.52W		9.3775
8	NW 53¼	11.60	N 6.94 W 4.64	18.16W 22.80W	126.0304	
9	NE 36¾	19.20	N 15.38 E 5.74	17.06W 11.32W	262.3828	
10	NE 22½	14.00	N 12.93 E 2.68	8.64W 5.96W	111.7152	
11	SE 76¾	12.00	S 2.75 E 5.84	0.12W 5.72 E		0.3300
12	SW 15	10.85	S 10.48 W 1.40	4.32 E 2.92 E	45.2736	
13	SW 16¾	10.12	S 9.69 W 1.46	1.46 E 0.00	14.1474	
					1285.1012 178.0499	178.0499
	Content as before in Chains				1107.0513	

It is needleſs here to inſert the Columns of Bearing or Diſtances in Chains, being the ſame as before.

No. Sta.	Latit. and Half Dep.	Mer. Diſt.	N Area.	S Area.
1	N 3.54 E 6.61	6.61 E 13.22 E	23.3994	
2	N 9.65 E 1.80	15.02 E 16.82 E	144.9430	
3	0.00 E 8.10	24.92 E 33.02 E		
4	S 29.44 W 9.74	23.28 E 13.54 E		685.3632
5	S 3.87 W 7.76	5.78 E 1.98W		22.3686
6	N 9.00 0.00	1.98W 1.98W		17.8200
7	S. 1.21 W 5.77	7.75W 13.52W	9.3775	
8	N 6.94 W 4.64	18.16W 22.80W		126.0304
9	N 15.38 E 5.74	17.06W 11.32W		262.3828
10	N 12.93 E 2.68	8.64W 5.96W		111.7152
11	S 2.75 E 5.84	0.12W 5.72 E	0.3300	
12	S 10.48 W 1.40	4.32 E 2.92 E		45.2736
13	S. 9.69 W 1.46	1.46 E 0.00		14.1474
			178.0499	1285.1012 178.0499
			Area in Chains as before	1107.0513

The Conſtruction of the Map from either the 3d or 4th Tables.

Plate X. Fig. 3.

Draw the Line NS. for a North and South Line, which call the firſt Meridian; in this Line aſſume any Point, as 1, for the firſt Station. Set the Northing of that ſtationary Line which is 3.54 from 1 to 2, on the ſaid Meridian Line. Upon the Point 2 raiſe a Perpendicular to the Eaſtward, the Meridian Diſtance being Eaſterly, and upon it ſet 13.22, the ſecond Number in the Column of Meridian Diſtance, from 2 to 2, and draw the Line 1 2, for the firſt Diſtance Line: From 2 upon the firſt Meridian, ſet the Northing of the ſecond ſtationary Line, that is, 9.65 to 3, and on the Point 3 erect a Perpendicular Eaſtward, upon which ſet the Meridian Diſtance of the ſecond Station 16.82, from 3 to 3, and draw the Line 2 3, for the Diſtance Line of the ſecond Station. And ſince the third Station has neither Northing nor Southing, ſet the Meridian Diſtance of it 33.02, from 3 to 4, for the Diſtance Line of the third Station. To the fourth Station there is 29.44, ſouthing which ſet from 3 to 5; upon the Point 5 erect the Perpendicular 5 5, on which lay 13.54, and draw the Line 4 5.

In the like Manner proceed to ſet the Northings and Southings on the firſt Meridian, and the Meridian Diſtances upon the Perpendiculars raiſed to the Eaſt or Weſt; the Extremities of which connected by Right Lines, will compleat the Map.

A Map may be in like Manner conſtructed from the firſt or ſecond Tables.

A TABLE

A TABLE *of Difference of Latitude and Half Departure, or of Northing, Southing, Half Eaſting, and Half Weſting, to every Quarter Degree of the Compaſs; the Diſtance being ſuppoſed to be One Four-Pole Chain.*

Deg.	Qrs.	N. S.	E. W.	Deg.	Qrs.	N. S.	E. W.
0.		1.0000	.0000	6.	0	.9945	.0522
	1	1.0000	.0022		1	.9940	.0544
	2	1.0000	.0044		2	.9936	.0566
	3	.9999	.0065		3	.9931	.0587
1.	0	.9998	.0087	7.	0	.9925	.0609
	1	.9997	.0109		1	.9920	.0631
	2	.9996	.0131		2	.9914	.0652
	3	.9995	.0153		3	.9909	.0674
2.	0	.9994	.0175	8.	0	.9903	.0696
	1	.9992	.0196		1	.9896	.0717
	2	.9990	.0218		2	.9890	.0739
	3	.9988	.0240		3	.9883	.0760
3.	0	.9986	.0262	9.	0	.9877	.0782
	1	.9984	.0283		1	.9870	.0804
	2	.9981	.0305		2	.9863	.0825
	3	.9978	.0327		3	.9855	.0847
4.	0	.9975	.0349	10.	0	.9848	.0868
	1	.9972	.0370		1	.9840	.0889
	2	.9969	.0392		2	.9832	.0911
	3	.9966	.0414		3	.9824	.0933
5.	0	.9662	.0436	11.	0	.9816	.0954
	1	.9958	.0457		1	.9808	.0975
	2	.9954	.0479		2	.9799	.0997
	3	.9949	.0501		3	.9790	.1018

Deg.	Qrs.	N. S.	E. W.	Deg.	Qrs.	N. S.	E. W.
12.	0	.9781	.1039	21.	0	.9336	.1792
	1	.9772	.1061		1	.9320	.1812
	2	.9763	.1082		2	.9304	.1832
	3	.9753	.1103		3	.9288	.1852
13.	0	.9744	.1124	22.	0	.9272	.1873
	1	.9734	.1146		1	.9255	.1893
	2	.9724	.1167		2	.9239	.1913
	3	.9713	.1188		3	.9222	.1933
14.	0	.9703	.1209	23.	0	.9205	.1953
	1	.9692	.1231		1	.9188	.1973
	2	.9681	.1252		2	.9171	.1993
	3	.9670	.1273		3	.9153	.2013
15.	0	.9659	.1294	24.	0	.9135	.2033
	1	.9647	.1315		1	.9117	.2053
	2	.9636	.1336		2	.9099	.2073
	3	.9624	.1357		3	.9081	.2093
16.	0	.9613	.1378	25.	0	.9063	.2113
	1	.9600	.1399		1	.9044	.2133
	2	.9588	.1420		2	.9026	.2152
	3	.9576	.1441		3	.9007	.2172
17.	0	.9563	.1462	26.	0	.8988	.2192
	1	.9550	.1482		1	.8968	.2211
	2	.9537	.1503		2	.8949	.2230
	3	.9524	.1524		3	.8929	.2250
18.	0	.9510	.1545	27.	0	.8910	.2270
	1	.9497	.1566		1	.8890	.2289
	2	.9483	.1586		2	.8870	.2308
	3	.9469	.1607		3	.8850	.2328
19.	0	.9455	.1627	28.	0	.8829	.2347
	1	.9441	.1648		1	.8809	.2366
	2	.9426	.1669		2	.8788	.2385
	3	.9412	.1689		3	.8767	.2405
20.	0	.9397	.1710	29.	0	.8746	.2424
	1	.9382	.1730		1	.8725	.2443
	2	.9366	.1751		2	.8703	.2462
	3	.9351	.1771		3	.8682	.2481

Deg.	Qrs.	N. S.	E. W.	Deg.	Qrs.	N. S.	E. W.
30.	0	.8660	.2500	39.	0	.7771	.3146
	1	.8638	.2519		1	.7744	.3163
	2	.8616	.2537		2	.7716	.3180
	3	.8594	.2556		3	.7688	.3197
31.	0	.8572	.2575	40.	0	.7660	.3214
	1	.8549	.2594		1	.7632	.3230
	2	.8527	.2612		2	.7604	.3247
	3	.8503	.2631		3	.7575	.3263
32.	0	.8480	.2649	41.	0	.7547	.3280
	1	.8457	.2668		1	.7518	.3296
	2	.8434	.2686		2	.7489	.3313
	3	.8410	.2705		3	.7460	.3329
33.	0	.8387	.2723	42.	0	.7431	.3345
	1	.8363	.2741		1	.7402	.3361
	2	.8339	.2759		2	.7373	.3378
	3	.8315	.2778		3	.7343	.3394
34.	0	.8290	.2796	43.	0	.7313	.3410
	1	.8266	.2814		1	.7282	.3426
	2	.8241	.2832		2	.7254	.3441
	3	.8216	.2850		3	.7224	.3457
35.	0	.8191	.2867	44.	0	.7192	.3473
	1	.8166	.2885		1	.7163	.3489
	2	.8141	.2903		2	.7132	.3504
	3	.8116	.2921		3	.7102	.3520
36.	0	.8090	.2939	45.	0	.7071	.3535
	1	.8064	.2956		1	.7041	.3551
	2	.8038	.2974		2	.7009	.3566
	3	.8012	.2991		3	.6978	.3581
37.	0	.7986	.3009	46.	0	.6946	.3596
	1	.7960	.3026		1	.6915	.3612
	2	.7933	.3044		2	.6883	.3627
	3	.7907	.3061		3	.6852	.3642
38.	0	.7880	.3078	47.	0	.6820	.3656
	1	.7853	.3095		1	.6788	.3671
	2	.7826	.3112		2	.6756	.3686
	3	.7799	.3129		3	.6724	.3701

Deg.	Qrs.	N. S.	E. W.	Deg.	Qrs.	N. S.	E. W.
48.	0	.6691	.3715	57.	0	.5446	.4193
	1	.6659	.3730		1	.5408	.4205
	2	.6526	.3745		2	.5373	.4217
	3	.6593	.3759		3	.5336	.4228
49.	0	.6560	.3773	58.	0	.5299	.4240
	1	.6527	.3788		1	.5262	.4251
	2	.6494	.3802		2	.5225	.4263
	3	.6461	.3816		3	.5188	.4274
50.	0	.6428	.3830	59.	0	.5150	.4286
	1	.6394	.3844		1	.5113	.4297
	2	.6361	.3858		2	.5075	.4308
	3	.6327	.3872		3	.5038	.4319
51.	0	.6293	.3885	60.	0	.5000	.4330
	1	.6259	.3899		1	.4962	.4341
	2	.6225	.3913		2	.4924	.4351
	3	.6191	.3926		3	.4886	.4362
52.	0	.6157	.3940	61.	0	.4848	.4373
	1	.6122	.3953		1	.4810	.4383
	2	.6087	.3966		2	.4771	.4394
	3	.6052	.3980		3	.4733	.4404
53.	0	.6018	.3993	62.	0	.4695	.4414
	1	.5983	.4006		1	.4656	.4425
	2	.5948	.4019		2	.4617	.4435
	3	.5912	.4032		3	.4579	.4445
54.	0	.5878	.4045	63.	0	.4540	.4455
	1	.5842	.4058		1	.4501	.4464
	2	.5807	.4070		2	.4462	.4474
	3	.5771	.4083		3	.4423	.4484
55.	0	.5736	.4095	64.	0	.4383	.4494
	1	.5700	.4108		1	.4344	.4503
	2	.5664	.4120		2	.4305	.4513
	3	.5628	.4133		3	.4265	.4522
56.	0	.5592	.4145	65.	0	.4226	.4531
	1	.5555	.4157		1	.4186	.4540
	2	.5519	.4169		2	.4147	.4549
	3	.5483	.4181		3	.4107	.4558

Deg.	Qrs.	N. S.	E. W.	Deg.	Qrs.	N. S.	E. W.
66.	0	.4067	.4567	75.	0	.2588	.4829
	1	.4027	.4576		1	.2546	.4835
	2	.3987	.4585		2	.2504	.4840
	3	.3947	.4594		3	.2461	.4846
67.	0	.3907	.4602	76.	0	.2419	.4851
	1	.3867	.4611		1	.2377	.4856
	2	.3827	.4619		2	.2334	.4862
	3	.3786	.4627		3	.2292	.4867
68.	0	.3746	.4636	77.	0	.2249	.4872
	1	.3705	.4644		1	.2207	.4876
	2	.3665	.4652		2	.2164	.4881
	3	.3624	.4660		3	.2122	.4886
69.	0	.3584	.4668	78.	0	.2079	.4890
	1	.3543	.4676		1	.2036	.4895
	2	.3502	.4683		2	.1993	.4899
	3	.3461	.4691		3	.1951	.4904
70.	0	.3420	.4698	79.	0	.1908	4908
	1	.3379	.4706		1	.1865	.4912
	2	.3338	.4713		2	.1822	.4916
	3	.3297	.4720		3	.1779	.4920
71.	0	.3255	.4727	80.	0	.1736	.4924
	1	.3214	.4734		1	.1693	.4927
	2	.3173	.4741		2	.1650	.4931
	3	.3131	.4748		3	.1607	.4935
72.	0	.3090	.4755	81.	0	.1564	.4938
	1	.3048	.4762		1	.1521	.4941
	2	.3007	.4768		2	.1478	.4945
	3	.2965	.4775		3	.1435	.4948
73.	0	.2924	.4781	82.	0	.1392	.4951
	1	.2882	.4788		1	.1348	.4954
	2	.2840	.4794		2	.1305	.4957
	3	.2798	.4800		3	.1262	.4960
74.	0	.2756	.4806	83.	0	.1218	.4962
	1	.2714	.4812		1	.1175	.4965
	2	.2672	.4818		2	.1132	.4968
	3	.2630	.4823		3	.1088	.4970

Deg.	Qrs.	N. S.	E. W.	Deg.	Qrs.	N. S.	E. W.
84.	0	.1045	.4972	87.	0	.0523	.4993
	1	.1002	.4974		1	.0480	.4994
	2	.0958	.4977		2	.0436	.4995
	3	.0915	.4979		3	.0392	.4996
85.	0	.0871	.4981	88.	0	.0349	.4997
	1	.0828	.4983		1	.0305	.4997
	2	.0784	.4984		2	.0262	.4998
	3	.0741	.4986		3	.0218	.4998
86.	0	.0697	.4988	89.	0	.0174	.4999
	1	.0654	.4989		1	.0131	.4999
	2	.0610	.4990		2	.0087	.5000
	3	.0566	.4992		3	.0043	.5000
				90.	0	.0000	.5000

A TABLE

A TABLE *for changing Degrees taken by the Circumferentor to thoſe of the Quartered Compaſs, and the contrary.*

Degrees		Degrees		Degrees		Degrees	
Cir.	Q.C.	Cir.	Q.C.	Cir.	Q.C.	Cir.	Q.C
360	Nor.		NW.		NW.		S.W
1	NW. 1	31	31	61	61	91	89
2	2	32	32	62	62	92	88
3	3	33	33	63	63	93	87
4	4	34	34	64	64	94	86
5	5	35	35	65	65	95	85
6	6	36	36	66	66	96	84
7	7	37	37	67	67	97	83
8	8	38	38	68	68	98	82
9	9	39	39	69	69	99	81
10	10	40	40	70	70	100	80
11	11	41	41	71	71	101	79
12	12	42	42	72	72	102	78
13	13	43	43	73	73	103	77
14	14	44	44	74	74	104	76
15	15	45	45	75	75	105	75
16	16	46	46	76	76	106	74
17	17	47	47	77	77	107	73
18	18	48	48	78	78	108	72
19	19	49	49	79	79	109	71
20	20	50	50	80	80	110	70
21	21	51	51	81	81	111	69
22	22	52	52	82	82	112	68
23	23	53	53	83	83	113	67
24	24	54	54	84	84	114	66
25	25	55	55	85	85	115	65
26	26	56	56	86	86	116	64
27	27	57	57	87	87	117	63
28	28	58	58	88	88	118	62
29	29	59	59	89	89	119	61
30	30	60	60	90	Weſt	120	60

Degrees		Degrees		Degrees		Degrees	
Cir.	Q. C.	Cir.	Q. C.	Cir.	Q. C.	Cir.	Q. C.
	S. W.		S. W.		S. E.		S. E.
121	59	151	29	181	1	211	31
122	58	152	28	182	2	212	32
123	57	153	27	183	3	213	33
124	56	154	26	184	4	214	34
125	55	155	25	185	5	215	35
126	54	156	24	186	6	216	36
127	53	157	23	187	7	217	37
128	52	158	22	188	8	218	38
129	51	159	21	189	9	219	39
130	50	160	20	190	10	220	40
131	49	161	19	191	11	221	41
132	48	162	18	192	12	222	42
133	47	163	17	193	13	223	43
134	46	164	16	194	14	224	44
135	45	165	15	195	15	225	45
136	44	166	14	196	16	226	46
137	43	167	13	197	17	227	47
138	42	168	12	198	18	228	48
139	41	169	11	199	19	229	49
140	40	170	10	200	20	230	50
141	39	171	9	201	21	231	51
142	38	172	8	202	22	232	52
143	37	173	7	203	23	233	53
144	36	174	6	204	24	234	54
145	35	175	5	205	25	235	55
146	34	176	4	206	26	236	56
147	32	177	3	207	27	237	57
148	33	178	2	208	28	238	58
149	31	179	1	209	29	239	59
150	30	180	Sou.	210	30	240	60

Degrees		Degrees		Degrees		Degrees	
Cir.	Q.C.	Cir.	Q.C.	Cir.	Q.C.	Cir.	Q.C.
	S. E.		S.W.		N.E.		N.E.
241	61	271	89	301	59	331	29
242	62	272	88	302	58	332	28
243	63	273	87	303	57	333	27
244	64	274	86	304	56	334	26
245	65	275	85	305	55	335	25
246	66	276	84	306	54	336	24
247	67	277	83	307	53	337	23
248	68	278	82	308	52	338	22
249	69	279	81	309	51	339	21
250	70	280	80	310	50	340	20
251	71	281	79	311	49	341	19
252	72	282	78	312	48	342	18
253	73	283	77	313	47	343	17
254	74	284	76	314	46	344	16
255	75	285	75	315	45	345	15
256	76	286	74	316	44	346	14
257	77	287	73	317	43	347	13
258	78	288	72	318	42	348	12
259	79	289	71	319	41	349	11
260	80	290	70	320	40	350	10
261	81	291	69	321	39	351	9
262	82	292	68	322	38	352	8
263	83	293	67	323	37	353	7
264	84	294	66	324	36	354	6
265	85	295	65	325	35	355	5
266	86	296	64	326	34	356	4
267	87	297	63	327	33	357	3
268	88	298	62	328	32	358	2
269	89	299	61	329	31	359	1
270	East	300	60	330	30	360	Nor.

The above Table is ſo plain in itſelf that it needs no further Explanation than this, that Cir. at the Top of the Column ſtands for Circumferentor, and Q. C. for Quarter'd Compaſs.

To change Degrees of the Circumferentor to thoſe of the Quarter'd Compaſs, without the foregoing Table, and the contrary.

1. If the Degrees are leſs than 90, they are that Number in the NW Quadrant.

2. If the Degrees are between 90 and 180, take them from 180, and the Remainder is the Degrees in the SW Quadrant.

3. If the Degrees are between 180 and 270 take 180 therefrom, and the Reſidue is the Degrees in the SE Quadrant.

4. If the Degrees are between 270 and 360, take them from 360, the Remainder is the Degrees in the NE Quadrant.

5. 360 is N, 180 is S, 90 is W, and 270 is E.

EXAMPLE.

1. Change 56°½, taken by a Circumferentor, to Degrees of the Quarter'd Compaſs.

Anſwer NW 56½

2. Change 126¼ to Degrees of the Quarter'd-Compaſs.

180
126¼

Anſwer 53¾

3. Change 234° ¼ to Degrees of the Quarter'd-Comp. 180

Anſwer SE 54¼

4. Change 324° ¾ to Degrees of the Quarter'd-Compaſs.

360
324¾

Anſwer NE 35¼

The contrary of the former.

Again to change Degrees taken by the Quarter'd-Compaſs to thoſe of the Circumferentor.

1. If the Degrees are in the NW Quadrant, they are a like Number by the Circumferentor.

2. If the Degrees are in the SW Quadrant, take them from 180, for the Degrees by the Circumferentor.

3. If the Degrees are in the SE Quadrant, add them to 180, their Sum will be thoſe for the Circumferentor.

4. If the Degrees are in the NE Quadrant, take them from 360, the Remainder will be thoſe for the Circumferentor.

5. N is 360, S 180, W 90, and E 270 Degrees.

This is ſo plain it requires no Example.

Having the Angles of the Field taken by a Theodolite, Semicircle, or Plane Table given, to reduce them to Angles from the Meridian, or to thoſe taken by the Circumferentor.

THE Angles of the Field, with the Bearing of the firſt, or any other ſtationary Line from the Meridian being given; the Bearings of all the other ſtationary Lines may be found by theſe two Rules.

RULE I.

If the Angle of the Field at any Station be more than 180, take 180 from it, and add the Remainder to the Bearing at the foregoing Station; the Sum if leſs than 360 will be the Bearing at the preſent Station or of the next Line: But if the Sum be more than 360, take 360 from it, and the Remainder will be the preſent Bearing.

RULE II.

If the Angle of the Field be leſs than 180, take it from 180, and from the Bearing at the foregoing Station take the Remainder, and you will have the Bearing of the preſent one: But if the Bearing at the foregoing Station be leſs than the firſt Remainder, to this foregoing Bearing add 360, and from that Sum take the firſt Remainder, and this laſt Remainder will be the preſent Bearing. An Example will render this Matter familiar and eaſy.

Required to change the following Angles of the Field, to Bearings of the Needle from the North, the Bearing of the firſt ſtationary Line being 262.

No. Sta.	Ang Field	The Manner of finding the Bearings.	Bearings.	No. Sta.
1	159			
2	200	200—180=20, 262+20=	282	2
3	270	270—180=90, 282+90=372, 372—360=	12	3
4	80	180—80=100, 12+360=372, 372—100=	272	4
5	98	180—98=82, 272—82=	190	5
6	100	180—100=80, 190—80=	110	6
	230	230—180=50, 50+110=	160	7
8	90	180—90=90, 160—90=	70	8
9	82	180—82=98, (70—82)=360+70—98=	332	9
10	191	191—180=11, 11+332=	343	10
11	120	180—120=60, 343—60=	283	11
		180—159=21, 283—21=	262	1
	1620	Sum of the Angles of the Field.		

The laſt found Bearing being the ſame as that taken at the firſt Station by the Needle, ſhews that all the Angles of the Field were truly taken.

The like we may alſo be confirmed of by adding up the Angles of the Field; for if their Sum be equal to twice as many right Angles as there are Stations or Sides in the Figure leſs by 4, they are truly taken. By Scholium, P. 157.

22 twice the Number of Stations.
4

18 Right Angles
90

1620 Sum of the Angles

Having found the Bearings from the North, or by the Circumferentor, the Bearings by the Quarter'd-Compaſs are eaſily obtained as before. And thus a Map can be more accurately protracted than from the Angles of the Field, and it's Area may be had by Calculation.

If you have all the Angles of the Field without any Bearing given, you may in the like Manner reduce them to Bearings, which will anſwer for Protraction by Parallels, or for finding the Area by Calculation; by ſuppoſing the Bearing of the firſt ſtationary Line to be any Number leſs than 360, and thence you may alſo know, if the Angles of the Field be truly taken. For though the Bearings of the ſeveral ſtationary Lines be falſe ones, yet the Map will be equal in all Reſpects to one protracted from Bearings taken by the Needle, and therefore the Area of each will be the ſame: But you will be at a loſs for the Situation of the Land ſurveyed, with reſpect to the North, South, Eaſt, and Weſt Points of the Compaſs.

The foregoing Rules demonſtrated.

Plate XI. Fig. 1.

Let 1, 2, 3, 4, 5, *&c.* repreſent a Plot of Ground, and let the Lines NS paſſing through every Station repreſent Meridian Lines.

The Arc NSo at the firſt Station, will be the Bearing of the Line 1 2, from the North, as if taken by the Circumferentor, which Arc is equal to the Arc Nr Sq at the ſecond Station, made by producing the Line 1 2, from the Parallel Poſition of the Needle. And the Arc rSqo at the ſecond Station,—rSq which is a Semicircle, is = oq, but NrSq, which was ſhewn to be equal to the Bearing at the firſt Station + oq = NrSqo, which will be the Bearing of the ſecond ſtationary Line.

Thus if from rSqo 200°, the Angle of the Field at the ſecond Station, rSq = 180° be taken, the Remainder oq will be 20°. But NrSq = 262° the Bearing of the firſt ſtationary Line + oq, which is 20° = NrSoq = 282°, the Bearing of the ſecond ſtationary Line.

Again at the third Station NrSq = NrSqo, the Bearing of the ſecond ſtationary Line, and rSqNo the Angle of the Field at the third Station; if then from it the Semicircle rSq be taken, the Remainder will be qNo, to which if NrSqo be added, the Sum will be the entire Circle + No, therefore if the Circle, or 360, be taken therefrom, the Remainder No will be the Bearing of the third ſtationary Line, *&c.* Q. E. D.

All the reſt will be manifeſt with little Conſideration when every Line is continued, and the Circles drawn as in the Scheme.

SECT. VI.

Containing the Nature of Off-ſets and Interſections; the Methods of enlarging, diminiſhing, and connecting Maps, with the Method of tracing the Down (or any other) Surveys. Alſo the Variation of the Compaſs and it's Uſes in Surveying. How to reduce one Meaſure to another in ſix Problems, with three other very uſeful Problems; and the whole concludes with ſome neceſſary Directions concerning Surveys in general.

Of OFF-SETS.

IN taking Surveys it is unneceſſary and unuſual to make a Station at every angular Point, becauſe the Field Work can be taken with much greater Expedition, by uſing Off-ſets and Interſections, and with equal Certainty.

Off-ſets are perpendicular Lines drawn or meaſured from the angular Points of the Land, that lie on the right or left Hand to the ſtationary Diſtance, thus.

Plate XI. Fig. 2.

Let the black Lines repreſent the Mearings of a Farm or Town-Land: And let 1 be the firſt Station, then if you have a good View to 2, you omit the angular Points between 1 and 2, and take the Bearing and Length of the ſtationary Line 1 2, and inſert them in your Field-Book: But in chaining from 1 to 2, ſtop at *d* oppoſite to the angular Point a, and in your Field-Book inſert the Diſtance from 1 to d, which admit to be 4C. 25L. as well as the Meaſure of the Off-ſet ad, which admit to be 1C. 12L. thus: By the Side of your Field-Book in a Line with the firſt Station ſay, at 4C. 25L. 1C. 12L. that is, at 4C. 25L. there is an Off-ſet to the left Hand of 1C. 12L.

This done, proceed on your diſtance Line to e, oppoſite to the Angle b, and meaſure eb, ſuppoſing then 1e to be 7C. 40L. and eb 3C. 40L. Say (ſtill in a Line with the firſt Station in your Field-Book) at 7C. 40L. L3C. 40L. that is at 7C. 40L. there is an Off-ſet to the left of 3C. 40L. proceed then with your diſtance Line to f, oppoſite to the Angle c, and meaſure fc; ſuppoſe then 1f to be 13C. and fc 1C 25L. ſay in the ſame Line as before, at 13C. L1C. 25L. Then proceed from f to 2, and you will have the Meaſure of the entire ſtationary Line 1 2, which inſert in it's proper Column by the Bearing.

In taking Off-ſets, it is neceſſary to have a Perch Chain, or a Staff of half a Perch, divided into Links for meaſuring them; for by theſe Means the Chain in the ſtationary Line is undiſturbed, and the Number of Chains and Links in that Line from whence, or to which, the Off-ſets are taken may be readily known.

Having arrived at the ſecond Station, if you find your View will carry you to 3, take the Bearing from 2 to 3, and in meaſuring the Diſtance Line, ſtop at l oppoſite to g; admit 2l to be 4C. 10L. and the Off-ſet lg 1C. 20L. then in a Line with the ſecond Station in your Field-Book, ſay at 4C. 10L. R 1C. 20L. that is the Off-ſet is a right Hand one of 1C. 20L. Again at m, which ſuppoſe to be 10C. 25L. from 2; take the Off-ſet mh of 1C. 15L. and in a Line with the ſecond Station ſay, at 10C. 25L. R 1C. 15L. In the ſame Line when you come to the Mearing at *i*, inſert the Diſtance 2*i*, 13C. 10L. thus, at 13C. 10L.0; that is at 13C. 10L. there is no Off-ſet. At *n*, which is 15C. from 2, take the Off-ſet *nk* 45L. and ſtill oppoſite to the ſecond Station ſay at 15C. L 45L.

Let the Line 36 repreſent the Mearing, which by Means of Water, Bryars, or any other Impediment cannot be meaſured. In this Caſe make one or more Stations within or without the Land, where the Diſtances may be meaſured, and draw a Line from the Beginning of the firſt to the End of the laſt Diſtance thus: Make Stations at 3. 4, and 5, taking the Bearings, and meaſuring the Diſtances as uſual. which inſert in your Field-Book, and draw a Mark like one Side of a Parentheſis from the third to the fifth Station, to ſhew that a Line drawn from the third Station to the fartheſt End of the fifth ſtationary Line will expreſs the Mearing. Thus,

	No. Sta.	Deg.	C. L.
(	3	172¼	5.45
	4	200	13.25
	5	250	3.36

Suppoſe the Point *p* of the Mearing to be inacceſſible, by Means of the Lines 6*p* or *p*7, being overflowed, or that a Quarry, Furze, *&c.* might prevent you taking their Lengths: In this Caſe take the Bearing of the Line 67, which inſert oppoſite to the ſixth Station in your Field-Book with the other Bearings; then direct the Index to the Point *p*, and inſert it's Bearing on the left Side of the Field-Book, oppoſite to the ſixth Station, annexing thereto the Words, *Int. for Mearing*; and having meaſured and inſerted the Diſtance 6 7, ſet the Index in the Direction of the Line 7*p*, and inſert it's Bearing on the left of the ſeventh Station of the Field-Book, annexing thereto, the Words *Int. for Mearing*; the Croſſing or Interſection of theſe two Bearings will determine the Point *p*, and of courſe the Mearing 6*p*7 is alſo determined.

If your View will then reach to the firſt Station, take it's Bearing, ſtationary Line, and Off-ſets, as before, and you have the Field-Book compleated. Thus,

The FIELD-BOOK.

Remarks and Interſect.	No. Sta.	Deg.	C. L.	OFF-SETS.
318 Int. to a Tower	1	358	22.12	*At* 4*C*. 25*L*. *L* 1*C*. 12*L*. *at* 7*C*. 40*L* *L* 3*C*. 40*L*. *at* 13*C*. *L* 1*C*: 25*L*.
231½ Int. to Ditto.	2	297¾	21.12	*At* 4*C*. 10*L*. *R* 1*C*. 20*L*. *at* 10*C*. 25*L*. *R* 1*C*. 15*L*. *at* 13*C*. 10*L*. 0: *at* 15*C*. *L*. 45*L*.
(	3	172¼	5.45	
	4	200	13.25	
	5	250	3.36	
155½ Int. for Mearing	6	125	15.15	
274 Int. for Mearing	7	105¼	15.10	*At* 1*C*. 20*L*. *L* 2*C*. 20*L* : *at* 7*C*. 45*L*. *L* 2*C*. 32*L*. *at* 11*C*. 25*L*. 0 *at* 12*C*. 25*L*. *R* 36*L*.

Cloſe at the firſt Station.

If you would lay down a Tower, Houſe, or any other remarkable Object in it's proper Place; from any two Stations take Bearings to the Object, and their Interſection will determine the Place where you are to inſert it, in the Manner that the Tower is ſet out in the Figure, from the Interſections taken at the firſt and ſecond Stations of the above Field-Book.

A Protraction of this will render all plain, on which lay off your Off-ſets and Interſections, and proceed to find the Content by any of the Methods in Section the 4th.

The foregoing Field-Book may be otherwiſe kept thus:

Remarks and Interſection	No. Stat.	Deg.	L-Han. Off-ſet C. L.	Diſ. C. L.	R. Han. Off-ſet C. L.
318 Int. to a Tower	1	358	1.12	4.25	
			3.40	7.40	
			1.25	13.00	
				22.12	
231½ Int. for Ditto	2	297¾		4.10	1.20
				10.25	1.15
				13.10	
			0.45	15.00	
				21.12	
	3	172½		5.45	
	4	200		13.25	
	5	250		3.36	
155½ Int. for Mearings	6	125		15.15	
274 Int. for Mearing	7	105	2.20	1.20	
			2.32	7.45	
				11.25	
				12.25	0.36
				15.10	

Cloſe at the firſt Station.

How

How to caſt up Off-ſets by the Pen.

Plate XI. Fig. 2.

$12 - 1f = 2f$, $1f - 1e = fe$, $1e - 1d = ed$.

Then $1d \times \frac{1}{2}da = 1da$, by Prob. 6, Page 183, and $\frac{1}{2}\,ed \times da+eb = adeb$ by Lemma, Page 226, alſo $\frac{1}{2}\,fe \times eb+fc = befc$, and $2f \times \frac{1}{2}\,fc = cf2$; the Sum of all which will be $1abc21$; the Area contained between the ſtationary Line 12, and the Mearing, $1\,abc\,2$.

In the ſame Manner you may find the Area of $2ibg2$ of $ik3i$, as well as what is without and withinſide of the Stationary Line 71.

If therefore the left Hand Off-ſets exceed the right Hand ones, it is plain, the Exceſs muſt be added to the Area within the ſtationary Lines, but if the right Hand Off-ſets exceed the left-Hand ones, the Difference muſt be deducted from the ſaid Area; if the Ground be kept on the right Hand as we have all along ſuppoſed; or in Words thus.

To find the Contents of Off-ſets.

1. From the Diſtance Line, take the Diſtance to the preceding Off-ſet, and from that the Diſtance of the one preceding it, *&c.* in Four-Pole Chains; ſo will you have the reſpective Diſtances from Off-ſet to Off-ſet, but in a retrogade Order.

2. Multiply the laſt of theſe Remainders by ½ the firſt Off-ſet, the next by ½ the Sum of the

firſt and ſecond, the next by half the Sum of the ſecond and third, the next by half the Sum of the third and fourth, *&c.* The Sum of theſe will be the Area produced by the Off-ſets.

Thus, in the foregoing Field-Book, the firſt ſtationary Line is 22C. 12L. or 11C. 12 L. of 4 Pole Chains. See the Figure.

	C. L.	C. L.	C. L.
From	11.12 = 1,2	6.50 = 1*f*	3.90 = 1*e*
Take	6.50 = 1*f*	3.90 = 1*e*	2.25 = 1*d*
	4.62 = 2*f*	2.60 = *ef*	1.65 = *ed*

C. L.
1*d* = 2.25 x 32L. half the firſt Off-ſet = .7200
ed = 1.65 x 1C. 26L. ½ the Sum of the 1st&2d 2.0790
ef = 2.60 x 1C. 32L. ½ the Sum of 2d & 3d = 3.4320
2*f* = 4.62 x 37L. half the laſt Off-ſet = 1.7094
Content of left Off-ſets on the firſt Diſt.
in Square Four-pole Chains 7.9404

In like Manner the reſt are perform'd.

The Sum of the left Hand Off-ſet will be 14.0856
And the Sum of the right Hand ones 3.6825

Exceſs of left Hand Off-ſets in Squ. 4PoleC. 10.4031

Acres 1.04031

.16124
4

Perch 6.4496

Exceſs of left Hand Off-ſets above the right Hand ones, 1A. 0R. 6P. to be added to the Area within the ſtationary Lines.

Of

Of INTERSECTIONS.

How to find the Area of a Piece of Ground by Interſections only, when all the Angles of the Field can be ſeen from any two Stations on the out Side of the Ground.

Plate XII. Fig. 1.

LET ABCDEFGA be a Field, H and I two Places on the out Side of it, from whence an Object at every Angle of the Field may be ſeen.

Take the Bearing and Diſtance between H and I, and ſet that at the Head of your Field-Book, as in the annexed one. Fix your Inſtrument at H, from whence take the Bearings of the ſeveral angular Points, A, B, C, D, *&c.* as they are here repreſented by the Lines HA, HB, HC, HD, *&c.* Again fix your Inſtrument at I, and take Bearings to the ſame angular Points, repreſented by the Lines IA. IB, IC, ID, *&c.* and let the firſt Bearings be entered in the ſecond Column, and the ſecond Bearings in the third Column of your Field-Book: Then it is plain that the Points of interſection, made from the Bearings in the ſecond and third Columns of every Line, will be the Angular Points of the Field or the Points A, B, C, D, *&c.* which Points being joined by right Lines, will give the Plan ABCDEFGA required.

Bea. 180 Diſ. 28C. of the Sta. H and I.

No.	Bear.	Bear.
A	261½	331½
B	265¾	317¼
C	248	307½
D	238¼	289
E	215½	262½
F	208½	286½
G	220	300

The ſame may be done from any two Stations within-ſide of the Land, from whence all the Angles of the Field can be ſeen.

This Method will be found uſeful in Caſe the ſtationary Diſtances from any Cauſe prove inacceſſible, or ſhould it be required to be done by one Party, when the other in whoſe Poſſeſſion it is, refuſes to admit you to go on the Land.

To find the Content of a Field by Calculation, which was taken by Interſection.

In the Triangle AIH, the Angles AHI, AIH, and the Baſe HI being known, the Perpendicular A*a*, and the Segments, of the Baſe *Ha*, *AI* may be obtained by Trigonometry: And in the ſame Manner all the other Perpendiculars B*b*, C*o*, D*d*, E*e*, *Ff*, G*g*, and the ſeveral Segments at *b*, *c*, *d*, *e*, *f* and *g*: If therefore the ſeveral Perpendiculars be ſuppoſed to be drawn in the Scheme, (which are here omitted to prevent Confuſion ariſing from a Multiplicity of Lines) it is plain that if from *b*BCDE*eb*, there be taken *b*BAGF*eb*, the Remainder will be the Map ABCDEFGA.

As before, Half the Sum of B*b*, and C*c*, multiplied by *bc*, will be the Area of the Trapezium *b*BC*c*; after the ſame Manner, Half the Sum of C*c*, and D*d*, multiplied by *cd*, will give the Area of the Trapezium *c*CD*d*; and again Half the Sum of D*d*, and E*e*, multiplied by *de*, gives the Area of the Trapezium *d*DE*e*; and the Sum of theſe three Trapezia will be the Area of the Figure *b*BCDE*eb*.

Again, in the ſame Manner, Half the Sum of *b*B, and *aA* multiplied by *ab*, will give the Area of the Trapezium *b*BA*a*; and Half the Sum of *a*A, and gG, by *ag*, gives the Trapezium *a*AG*g*; to theſe add the Trapezia *g*GF*f*, and *f*FE*e*, which are found in the like Manner, and you will have the Figure *b*BAGFE*eb*, and this taken from *b*BCDE*eb*, will leave the Map ABCDEFGA. Q. E. F.

It will be ſufficient to protract this Kind of Work, and from the Map to determine the Area, as well as in Plate X Fig. 3. to find the Areas of the Pieces 34563, and 6*p*76, from Geometrical Conſtructions.

How to determine the Station where a Fault has been committed in a Field-Book without the Trouble of going round the whole Ground a ſecond Time.

From every fourth or fifth Station if they be not very long ones, or oftner if they are, let an Interſection be taken to any Object, as to any particular

Part of a Caſtle, Houſe, or Cock of Hay, *&c.* or if all theſe be wanting, to a long Staff with a white Sheet or Napkin ſet thereon to render the Object more conſpicuous, and let this be placed on the Summit of the Land, and let the reſpective Interſections ſo taken be inſerted on the left Hand Side of the Field-Book, oppoſite to the Stations from whence they were reſpectively taken.

In your Protraction as you proceed, let every Interſection be laid off from the reſpective Stations from whence they were taken, and let theſe Lines be continued; if they all converge or meet in one Point we thence conclude all is right, or ſo far as they do converge; but if we find a Line of Interſection to diverge or fly off from the reſt, we may be ſure that either a Miſtake has happened between the Station, the foregoing Interſection was taken at, and the Station from whence the Interſection, Line diverges; or there muſt be an Error in the Interſection; but to be aſſured in which of theſe the Fault is, protract on to the next Interſection and having ſet it off, if it converges with the reſt, tho' the foregoing one did not, we may conclude the Fault was committed in taking the laſt Interſection but one, and none in any Station, and that ſo far is true as is protracted; but if this as well as the foregoing Interſection diverge, or fly from the Point of Concourſe or converging Point of the reſt, the Error muſt have its Riſe from ſome Station or Stations, at or after that, from whence the laſt converging Interſection Line was taken; ſo that by going to that Station on the Ground and proceeding on to that where the next, or from whence the following diverging Interſection was taken, we can readily and with but little Trouble ſet all to Rights.

But in moſt Tracts of Land, one Object cannot be ſeen from every Station, or from perhaps one fourth of them; in this Caſe we are under the neceſſity to move the Pole after we begin to loſe Sight of it to ſome other Part of the Land, where it may be ſeen from as many more Stations as poſſible; which is eaſily done by viewing the Mearing before it be ſurveyed: The Pole then being fixed in an advantageous Place, the firſt Interſection to it is beſt to be made from the ſame Station from whence the laſt one was taken, and then as often as may be thought convenient as before; in like Manner the whole may be done by the Removal of the Pole.

When we here ſpeak of Stations we do not mean ſuch as are uſually taken at every particular Angle of the Field: for it is to be apprehended, that every ſkilful Surveyor, particularly ſuch who uſe Calculation, will take the longeſt Diſtances poſſible, not only to leſſen the Number of Stations, for the Eaſe of either Protraction or Calculation, but with greater Certainty to account for the Land paſſed by, on the right Hand or on the left, which is taken by Off-ſets: And ſurely it will be allowed that any Meaſure taken on the Ground and the Content thence arithmetically computed, will be much more accurate than that which is obtained from any Geometrical Projection.

From what has been ſaid it is plain, that from this Method any Fault committed in a Survey can be readily determined, and therefore muſt be much preferable to the preſent Method of taking Diagonals, or the Bearings and Lengths of Lines acroſs Land, to accompliſh that End: which laſt Method is too frequently uſed by Surveyors to approximate or arrive near the Content, which will ever remain uncertain, let theſe Diagonals be ever ſo many, 'till the Station or Stations wherein the Error or Errors were committed, be found; and the Fault or Faults be corrected.

Where one Diagonal is taken it may perhaps cloſe or meet with one Part of the Survey and not with the other; in this Caſe, if the Surveyor would diſcover his Error, he muſt ſurvey that Part of the Land which did not cloſe, and this may be half or more, of the whole. And ſhould the Diagonal cloſe with neither Part, but be too long, or too ſhort, or ſhould it fall on either Side of the aſſigned Point it was to cloſe with, he ought to go over the whole, and make a new Survey of it in order to diſcover his Error.

A Number of Diagonals are frequently taken, the Sum of the Lengths of which very often exceeds the Circuit of the Ground, and after all they are but Approximations, and the Content remains uncertain as before; therefore, he who returns a Map, made up by the Aſſiſtance of Diagonals, where there remains a Miſcloſure in any one Part, runs the Rique of being detected in an Error, and muſt ſuffer Uneaſineſs in his Mind, as he cannot be certain of the Return he makes.

The frequent Miſcloſures which are botched up by Diagonals, occaſion the many and frequent ſcandalous Broils and Animoſities between Surveyors, which tend to the Loſs of Character of the one or the other, and indeed often to the Diſrepute of both, as well as to that of the Science they profeſs.

But theſe may be eaſily remedied by Interſections, and the Bearing or Line be adjuſted where the Fault was committed, and 'till this be found, nothing can be certain.

To enlarge or diminiſh MAPS.

How to enlarge, or diminiſh a Map, or how to reduce a Map from one Scale to another: Alſo the Manner of uniting ſeparate Maps of Lands which join each other, into one Map of any Aſſigned Size.

LAY the Map you would enlarge, over the Paper on which you would enlarge it, and with a fine protracting Pin, prick thro' every angular Point of your Map, join theſe Points on your Paper, (laying the Map you Copy before you) by pencilled or popped Lines, and you have the Copy of the Map you are to enlarge: In this Manner any Protraction may be copied on Paper, Vellum, or Parchment for a fair Map.

If you would enlarge a Map to a Scale which is double, or treble, or quadruple to that of the Map to be enlarged; the Paper you muſt provide for it's Enlargement muſt be two, or three, or four Times as long and broad as the Map; for which Purpoſe in large Things you will find it neceſſary to join ſeveral Sheets of Paper, and to cement them with white Wafer or Paſte, but the former is beſt.

Then pitch upon any Point in your copied Map, for a Center; from whence if Diſtances be taken to it's extream Points, and thence if thoſe Diſtances be ſet in a right Line with (but from) the Center, and theſe laſt Points fall within your Paper, the Map may be encreaſed on it to a Scale as large again as it's own; and if the like Diſtances be again ſet outwards in right Lines from the Center, and if theſe laſt Points fall within your Paper, it will contain a Map encreaſed to a Scale three Times as large as it's own, *&c.*

Plate XII. Fig. 2.

Let the pricked or popped Lines repreſent the Copy of a Down or old Survey, laid down by a Scale of 80 Perches to an Inch, and let it be required to enlarge it to one laid down by 40 to an Inch.

Pitch upon your Center as ⊙, from whence thro' *a* lay the fiducial Edge of a thin Ruler, with a fine pointed Pair of Compaſſes, take the Diſtance from *a* to the Center ⊙, and lay it by the Ruler's Edge from *a* to A: In the like Manner take the Diſtance from the next Station *b* to the Center ⊙, and lay it over in a right Line from *b* to B, and join the Points A and B by the right Line AB: In the like Manner ſet over the Diſtance from every Station to the Center, from that Station outwards, and you will have every Point to enlarge to; the joining of theſe conſtantly as you go on by right Lines, will give you the enlarged Map required.

In taking the Diſtances from every Station to the Center, ſet one Foot of the Compaſſes in the Station, and the other very lightly over the Center-Point, ſo lightly as ſcarcely to touch it, otherwiſe the Center-Point, will become ſo wide that it may occaſion ſeveral Errors in the enlarged Map: For if you err from the exact Center but a little, that Error will become double, or treble, or quadruple,

druple, as you enlarge to a Scale that is double or treble or quadruple, of the given one; therefore great Accuracy is required in enlarging a Map.

When you have done with a Station, give a Daſh with a Pen or Pencil to it, ſuch as at the Station *a* and *b*; by this Means you cannot be diſappointed in miſſing a Station, or in laying your Ruler over one Station twice.

From what has been ſaid it is plain, that if a Map is to be enlarged to one whoſe Scale is double the given one, that the Diſtances from the reſpective Stations to the Center, being ſet over by the Ruler's Edge, will give the Points for the enlarged one. And thus may a Map be enlarged from a Scale of 160 to one of 80, from one of 80 to one of 40, from one of 20 to one of 10 Perches to an Inch, *&c.* for to enlarge to a Scale that is double; the Number of Perches to an Inch, for the enlarged Map muſt be Half of thoſe to an Inch for that to be enlarged: To enlarge to a Scale that is treble the given one, the Number of Perches to an Inch for the enlarged Map, will be one third of thoſe for the other; if to a Scale that is quadruple the given one, the Number of Perches to an Inch, for the enlarged Map, will be one fourth of thoſe for the other, *&c.* therefore if you would enlarge a Map which is laid down by a Scale of 120 Perches to an Inch, to one of 40 Perches to an Inch; the Diſtances from the ſeveral Stations to the Center, being ſet twice beyond the ſaid Stations, will mark out the ſeveral Points required, for theſe Points will be three Times farther from the Center than the Stationary Points of the Map are.

In the ſame Manner, if you would enlarge a Map from a Scale of 160, to one of 40 Perches to the Center, being ſet three Times beyond ſaid Stations, will lay out the Points for your enlarged Map, for theſe Points will be four Times farther from the Center than are the Stations of the Map.

When a Map is enlarged to another, whoſe Scale is double or treble, or quadruple, *&c.* of the given one, every Line, as well as the Length and Breadth of the enlarged Map will be double, or treble, or quadruple, *&c.* thoſe of the given one, for it muſt be eaſy to conceive that thoſe Maps are like: But the Area, if the Scale be double, will be four Times; if treble, nine Times; if quadruple, ſixteen Times that of the given Figure; that is, it will contain four, nine, or ſixteen Times as many ſquare Inches as the given one (for it has been ſhewn that like Polygons are in a duplicate Proportion with their homologous Sides.) Yet theſe Figures being caſt up by their reſpective Scales, will produce the ſame Content.

Thus much is ſufficient for enlarging Maps, and from hence, diminiſhing of them will be obvious; for one fourth, one third, or half the Diſtances from the ſeveral Stations to the Center, will mark out Points, which, if joined will compoſe a Map ſimilar to the given one, whoſe Scale will be four Times, three Times, or twice as ſmall as the given one.

Thus, if we would reduce a Map from 40 to 80, from 20 to 40, from 10 to 20 Perches to an Inch, *&c.* half the Diſtance of the Stations from the Center will give Points requiſite for drawing the Map; if we would reduce from 40 to 120, from 20 to 60, from 10 to 30 Perches to an Inch, *&c.* one third of the Diſtances to the Center, will give the Points for the Map; And if we would reduce from 40 to 160, from 20 to 80, from 10 to 40 Perches to an Inch, *&c.* one fourth of the Diſtances to the Center, will give the Points for the Map.

By the Methods here laid down I have reduced a Map from a Scale of 40 to one of 20 Perches to an Inch, which contained upwards of 1200 Acres, and conſiſted of 224 ſeparate Diviſions, without the leaſt Confuſion from the Lines; for none can ariſe if the Methods here laid down be ſtrictly obſerved.

I have alſo from the ſame Methods reduced a large Book of Maps, each of which was an entire Skin of Parchment, and the whole contained upwards of 46000 Acres, to a Pocket Volume; and afterwards connected all theſe Maps into one Map, which was contained in one Skin of Parchment: Therefore upon the whole I do recommend theſe Methods for reducing Maps to be much more accurate than any of the Methods commonly uſed, ſuch as ſquaring of Paper, uſing a Parallelogram, proportional Compaſſes, or any other Method I ever met with, though the Figures to be reduced were ever ſo numerous, irregular, or complicated.

How to unite ſeparate Maps of Lands which join each other, into one Map of any aſſigned Size.

If there be ſeveral large Maps contained in a Book, each of which ſuppoſe to take up a Skin of Parchment, or a Sheet of the largeſt Paper; which Maps of Lands join each other; and it be required to reduce them to ſo ſmall a Scale, that all of them when joined together may be contained in one Skin, half a Skin, or any aſſigned ſized Piece of Parchment, or Paper.

Having pricked off and copied the ſeveral Maps on any Kind of Paper, unite them by cutting with Sciſſars along the Edge of one Mearing which is adjoining the other, but not cutting by the Edge of both, and throw aſide the Parts cut off; then lay theſe together on a large Table, or on the Floor, and where the Mearings agree, they will fit in with each other as Indentures do; and after this Manner they are eaſily connected: Meaſure then the Length and Breadth of the entire connected Maps, and the Length and Breadth of the Parchment or Paper you are confined to; if the former be three, four, or five Times greater, (that is longer and broader) than the latter, reduce each copied Map ſeverally to a Scale that is three, or four, or five Times leſs, as before; and the ſame Parts of the Mearings you cut by in the large Maps, by the ſame you muſt alſo cut in ſmall ones, and unite the ſmall as the large ones were united; cementing them together with white Wafer: Thus will your Map be reduced to the aſſigned Size, which copy over fair, on the Parchment, or Paper you were confined to.

But it is not always that a Perſon is confined to a given Area of Parchment, or Paper; in ſuch Caſes, if there are many large Maps to be united into one, reduce each of them ſeverally to a Scale of 160 Perches to an Inch, and unite thoſe by the Contiguity of Mearings as before: Or if you have a few, it will be ſufficient to reduce them to a Scale of 120, *&c.* But having the Maps given, and the Scale by which they are laid down, your Reaſon will be ſufficient to direct you to know, what Scale they ſhould be reduced to.

How to trace LANDS from the DOWN (or any other SURVEY.

IN the Surveyor-General of Land's Office (which is now kept in the ſame Building with the Ordnance-Office, in the lower *Caſtle-Yard*, *Dublin*) are kept what is called the Down Surveys, or Surveys of moſt of the forfeited Lands in this Kingdom. Theſe Surveys were done under the Direction of Sir *William Petty*, Bart. then Surveyor-General of Lands, and afterwards Earl of *Shelburn* about the Year 1641; there was a Survey previous to this, called the Civil Survey, or one by Eſtimation; and becauſe the latter of theſe was laid down by Maps on Paper, it thence got the Name of the Down Survey.

About forty Years ago the Treaſury-Office in *Eſſex-Street*, where the Surveyor-General's Office was alſo then kept, was burnt; and though moſt of the Down Surveys were ſaved, yet ſome were conſumed, and this is the Reaſon that the Surveys of all the forfeited Lands are not to be had at the Surveyor-General's Office; but Sir *Thomas Taylor*, Bart. has a fair and true Copy of all the Down Surveys of the forfeited Lands, which often proves of great Uſe to Gentlemen; though the Down Surveys only, are thoſe allowed by the Laws of the Land to be final and deciſive.

It will not here be unneceſſary to obſerve, that *6s. 8d.* is to be paid for the Copy of every Denomination, but nothing for a Search, if it be found; otherwiſe *5s.* is to be paid for the Search, if the Denomination cannot be found, that is, if it were not forfeited or burnt.

The Differences generally ariſing about Lands, wherein Down Surveys are found neceſſary, are for the moſt Part of this, or the like Nature.

A, who reſides in the Country upon his own Eſtate, takes Lands contiguous thereto from B, who reſides in the City or far from his Eſtate, and who perhaps never ſaw it. A in ſome Time defaces the contiguous Mearings, and runs others within ſide of B's Ground, ſometimes nearly ſimilar, and parallel to the defaced Mearing. At the Expiration of the Leaſe, A, or his Heir (who may be innocent of the Matter,) delivers to B, or his Heir, the Lands according to the new run Mearings; but the latter by a Survey he cauſes to be made, in order to ſet it to a new Tenant, finds his Land widely defective of the Number of Acres demiſed to A, which alarms him; and upon applying to A, without finding any Redreſs, a Law-Suit is commenced, and the Court orders a Trace to be run from the Lines of the Down Survey; If it is to be had, this readily diſcovers the Fraud, if not, the Matter is determined by the Evidences of the oldeſt Perſons of honeſt Repute in the Neighbourhood.

The beſt Way to avoid all future Diſputes concerning Lands, is to have the Lands Surveyed before

fore ſet, and to annex Maps thereof to the Leaſes. If afterwards the Mearings ſhould be defaced or changde, or any Diſpute ſhould ariſe concerning them; the Map to the Leaſe may be eaſily traced, and the Difference eaſily reconciled.

A Down Survey is thus Traced.

Take a Survey of the Land, as it is ſhewn you by the beſt Information you can receive, and make a Protraction thereof upon parallel Paper, by a Scale of 40 Perches to an Inch.

Then enlarge your Down Survey to a Scale of 40 as before, (provided it be not already laid down by that Scale, but they are oftner laid down by a Scale of 80, of 120, and ſometimes of 160 Perches to an Inch) for it is not ſafe to enlarge the Down Survey from any ſmall Scale to one larger than 40, becauſe the Errors of the Diſtances from the ſtationary Points if ever ſo minute, will be thereby much encreaſed.

I cannot omit here to obſerve, that ſome in order enlarge the Down Survey, have pricked it off on Parallel Paper, and after drawing the Map thereon, have (by producing every Diſtance Line) found their Bearings, and by meaſuring the Lines from the given Scale, have formed a Field-Book: And after all this Trouble, when they have protracted it to the aſſigned Scale, all was wrong, for it would not meet or cloſe: Of this I have ſeen many Inſtances; yet the Surveyor would make it do, as Miſcloſures are many Times forced to do; the Conſequence of either, the moſt unſkilful Surveyor cannot be ignorant of.

Let the Enlargement you make of the Down Survey to the like Scale of your Protraction, be upon the thineſt Paper you can get, which rub over with any Kind of Oil, to make it tranſparent, but wipe off the Oil that it may not ſmeer the Protraction. If Oil of Turpentine can be readily had, it will do beſt, becauſe it won't ſmeer; but in Caſes of Extremity, Butter turned to Oil will do.

Then apply your oiled Map, or Enlargement of the Down Survey, upon your protracted Map, and you will readily ſee what Points of the one will coinicde with thoſe of the other; and when you have brought the greateſt Number of Points in each to agree that is poſſible, the Maps are then applied to the greateſt Advantage: See then where the Lines of the Down Survey run within or without thoſe of your Protraction, and prick them on it; and to diſtinguiſh them from the black Lines of the Protraction, let them be drawn in red Ink, or popped.

Produce every ſtationary Line of the Down Survey, (or every red, or popped Line) forwards, that runs within or without Side of the Protraction; then if the Center of your Protractor be applied to every ſtationary Point of ſuch continued Lines, and your Protractor be kept Parallel to the Parallels or Meridians on the Protraction; the Continuation of every ſtationary Line, will point out on the Protractor's Edge, the Degree or Bearing of ſuch Lines: If theſe Bearings be inſerted in a Field-Book, as well as the Lengths of the ſeveral ſtationary Lines, (which are eaſily obtained by meaſuring them from the Scale by which they were laid down,) that Field-Book will direct your Trace.

Being thus prepared, go to the Ground and ſet up your Inſtrument at the firſt Station of ſaid Field-Book, and turn the Inſtrument about 'till you make the Needle (if you uſe a Circumferentor) point to the firſt Bearing in your Field-Book; (or if you uſe a Theodolite ſet it North and South by the Needle, and bring the Index to cut the ſaid Degree or Bearing): Let the Inſtrument ſtand in this Direction, 'till you ſend forth ſome Perſon to a Diſtance, greater if poſſible than the aſſigned one, or that of your ſtationary Line; then look through the Sights which govern the Direction, and waft your Hat to one or to the other Side, as a Token to the Perſon you ſent forward to ſtand more to the right or to the left, and continue to give the Signal 'till you have him in the Direction of the Sights, and then put on your Hat as a Signal to him that he is in the true Direction; there let him continue 'till you have meaſured the ſtationary Line in your Field-Book towards him, and at the End thereof cauſe a large hole to be dug, or a Stake to be driven, and your firſt Line is traced.

In like Manner proceed to trace every Diſtance Line, cauſing a large Hole to be made, or a Stake to be driven, or both, at every Station; 'till you have laid out and traced all your Field-Notes: And when Lines, or Ditches, are run from one Hole, or Stake to another, they will be the true Mearings, or Boundaries of the Land.

The Area comprehended between the popped, or red Lines, and the black ones of your Protraction, will be what is gained or loſt.

This is the beſt Method of tracing Down Surveys; and it is plain, that the ſame or a like Application may be made of any Survey whatſoever, ſo as to run a Trace thereof.

The

The Variation of the COMPASS;

And how to find it by Amplitudes or Azimuths of the Sun.

1. IT was before obſerved, that the Needle does not point truly to the North or South Points of the Horizon: The Number of Degrees therefore, that the Points of the Needle, are from the North or South Points of the Horizon, is called the *Variation of the Needle*, or *Compaſs*.

This Variation differs widely in many Places; for in ſome, the Needle will point ſeveral Degrees on the Weſt Side of the North; at others there will be little or no Variation, and again, at others it will point ſeveral Degrees on the Eaſt Side; in the ſame Place it differs ſenſibly in a few Years: The true Cauſe or Theory of which, has not hitherto been diſcovered or explained for Want of a ſufficient Number of Obſervations.

2. The Globe of the Earth revolves round it's Axis in twenty-four Hours from Weſt to Eaſt; and hence all Celeſtial Bodies ſeem to move from Eaſt to Weſt.

3. The Extremities of the Axis are called the *Poles*; the one the North or *Arctic*, and the other the South or *Antarctic*. And if the Axis be produced to the Heavens, it will point out the *Celeſtial Poles*.

4. If a Circle be ſuppoſed to paſs round the Globe of the Earth, ſo as to be equidiſtant from each Pole, it is called the *Equator*, or *Equinoctial Line*, and by ſome the *Line* only.

And if the Plane of the Equator be produced to the Heavens, it will lay out the *Celeſtial Equator*.

5. *The Latitude of any Place*, is it's neareſt Diſtance to, and counted from the Equator in Degrees and Minutes; and is North or South as it lies on the North or South Side of the Equator.

6. The Poles are 90 Degrees from the Equator; therefore the *Complement of the Latitude of any Place*, is the Latitude taken from 90 Degrees, or the Diſtance of the place from it's neareſt Pole.

7. *The Declination of the Sun*, is the neareſt Diſtance thereof from the celeſtial Equator counted in Degrees and Minutes; and is North or South, as it lies on the North or South Side of the Equator.

8. The Sun's Declination taken from 90, leaves the *Complement* thereof; or it's Diſtance from the neareſt celeſtial Pole.

9. *The Sun's Altitude*, is the Number of Degrees and Minutes the Sun is above the Horizon, and is eaſily found by a Quadrant as before.

10. What the Sun's Altitude wants of 90, or the Sun's Diſtance from the *Zenith* or Point of the Heavens perpendicularly over you, is *the Complement of the Altitude*.

11. *The Magnetical Amplitude*, is the Complement of the Sun's Bearing at Riſing or Setting, taken by the Quarter'd-Compaſs; or it is the Number of Degrees the Sun is from the Eaſt or Weſt Point of the Compaſs, at Riſing or Setting.

12. *The True Amplitude* is the Complement of Degrees the Sun would riſe or ſet on if the Compaſs did not vary; or it is the Number of Degrees the Sun is from the Eaſt or Weſt Point of the Horizon, at Riſing or Setting; and this true Amplitude is always North, if the Sun's Declination be North, or South if the Sun's Declination be South.

To find the Variation by the Amplitudes.

Having the Latitude of the Place, and the Sun's Declination given, the true Amplitude is found by this Aſtronomical Proportion, *viz.*

As the Co-ſine or Sine Complement of the Latitude,
Is to the Sine of the Sun's Declination,
So is Radius
To the Sine of the True Amplitude.

Then if both Amplitudes be North, or both South, their Difference is the Variation, but if one be North and the other South, their Sum is the Variation.

To know whether the Variation be Eaſterly or Weſterly.

Let the Obſerver turn his Face to the Sun, then if the true Amplitude be to the right Hand of the magnetical one, the Variation is Eaſterly, but if to the left, Weſterly.

EXAMPLE I.

On the 17th Day of *May* 1752, the Sun's Bearing at Riſing, being NE 71°, in the Latitude 53°.20' N. required the true Amplitude, and the Variation of the Needle.

Becauſe the annexed Table is for the *New-Stile*, which is 11 Days before the *Old*; we find the Sun's Declination for the 28th of *May* in the Leap Year (for this Table does not take Place before 1753) to be 21°.34' N.

After the ſame Manner this Table may ſerve 'till 1753.

To find the True Amplitude.

As the Co-ſine of the Latitude	36°.40'	9.77609
Is to the Sine of the Declination	21°. 34 N	9.56536
Radius	90 .00	10.00000
To the Sine of the true Ampl.	38 .00 N	9.78927

90°—71° = 19° the Mag. Amp. from the Eaſt.

True Amplitude	E	38°.00' N	for the Decl. is N.
Magnetical Ampl.	E	19 .00 N	
Variation		19 .00 W	becauſe the true

Amplitude is to the left of the magnetical.

EXAM-

EXAMPLE II.

Suppoſe the Sun's true Amplitude is found to be W 42°.00′ S, and the magnetical amplitude W 23. 00 S, the Sun's Bearing at ſetting being SW. 67°. Required the Variation.

90°—67° = 23° the magnetical Amplitude from the Weſt.

True Amplitude	W	42°.00′	S
Magnetical Amplitude	W	23 .00	S
Variation		19.00	W

In this Caſe alſo the true Amplitude is to the left of the magnetical; and therefore the Variation is weſterly.

EXAMPLE III.

Sun's Bearing at Riſing being SE 77°½, and the true Amplitude being found to be E 10. 12′ N. required the Variation.

90°—77°½ = 12°½ the magnetical Amplitude from the Eaſt.

True Amplitude	E	10°.12′	N
Magnetical Amplitude	E	12 .30	S
Variation		22.42	W

The true Amplitude being ſtill to the left, the Variation is weſterly.

EXAM-

EXAMPLE. IV.

Sun's Bearing at Setting is SW 81°½, and the true Amplitude is found to be W 6°. 16'. N. Required the Variation.

90°—81°½ = 8°½ the magnetical Amplitude from the Weſt.

True Amplitude	W 6°.16'.N
Magnetical Amplitude	W 8 .30 S
Variation	14.46 E

The true Amplitude being to the right of the magnetical, the Variation is eaſterly.

2. *To find the Variation by Azimuths.*

13. The Sun's magnetical Azimuth is the Bearing thereof at any Time of the Day, taken by the Quarter'd-Compaſs; that is counted from the North or South towards the Eaſt or Weſt Points of the Box.

14. The Sun's true Azimuth is the Point of the Compaſs it would bear from you upon, if there was no Variation: or it is the Diſtance intercepted between the North or South Points of the Horizon, and a vertical Circle, or Circle drawn from the Zenith through the Sun to the Horizon.

Having the Latitude of the Place, the Sun's Declination, and it's Altitude given, the true Azimuth is obtained by the following aſtronomical Proportions.

1. *As the Tangent of half the Complement of the Latitude*

Is to the Tangent of half the Sum of the Diſtance of the Sum from the Pole, and Complement of the Altitude,

So is the Tangent of half the Difference between the Diſtance of the Sun from the Pole, and Complement of the Altitude,

To the Tangent of a fourth Arc.

Add this fourth Arc and Half the Complement of the Latitude together, their Sum will give a fifth Arc; from which if the Complement of the Latitude be taken, the Remainder will give a ſixth Arc. Then ſay,

As Radius
Is to the Tangent of the Altitude,
So is the Tangent of the ſixth Arc
To the Co-ſine of the Sun's true Azimuth.

Which is counted from the North or South, to the Eaſt or Weſt, according to the Sun's Situation at the Time and Place of Obſervation.

If the Latitude of the Place and the Sun's Declination are both North or both South, the Declination taken from 90°, gives the Sun's Diſtance from the Pole; but if one be North and the other South, the Declination added to 90° will give the Sun's Diſtance from that Pole which is neareſt the Obſerver.

If both Azimuths are Eaſt or Weſt, their Difference is the Variation, but if one be Eaſt, and the other Weſt, their Sum is the Variation.

To know whether the Variation be Eaſterly or Weſterly.

Juſt as with the Amplitudes, let the Obſerver's Face be turned to the Sun; then if the true Azimuth be to the right Hand of the magnetical one, the Variation is Eaſterly, but if to the left, Weſterly.

EXAMPLE I.

In the Latitude 53° 20′ N, the Sun's Declination being 19°.03′ N, I find by Obſervation the Sun's Altitude to be 37°.30′ and it's Magnetical Azimuth to be SE 51°. Required the Variation.

90°—53°.20′ = 36°.40′ the Compt. of the Latitude
18.20, ½ the Compt of the Latitude
90°—37°.30′ = 52°.30′. the Compt. of the Altitude.

90°—19°.03′ = 70°.57′ the Sun's Diſ. from the Pole
52.30 Compt. of the Altitude

123.27 Sum

61.43 Half Sum

18.27 Difference

9.13 Half Difference

As

As the Tang. of ½ the Comp. of the Latitude,	18°.20ʹ—	9.52031
Is to the Tangent of ½ the Sum of the Diſtance of the Sun from the Pole and Complement of the Altitude,	61. 43 —	10.26916
: : Tang. of ½ their Difference	9. 13 —	9.21022
		19.47938
To Tangent of a 4th Arc	42. 18 —	9.95907

Half the Comp. of the Latitude	18°.20ʹ
The 4th Arc	42. 18
Their Sum is the 5th Arc	60. 38
Complement of the Lat. ſubtract	36. 40
Gives the 6th Arc	23. 58

As Radius	90°.00ʹ—	10.00000
Is to the Tang. of the Alt.	37. 30 —	9.88498
: : Tangent of the 6th Arc	23. 58 —	9.64790
Co-ſine of the Sun's true Azim.	70. 04 —	9.53288

True Azimuth	S	70°.04ʹ	E
Magnetical Azimuth	S	51. 00	E
Variation		19. 04	W

The true Azimuth being to the left of the magnetical one, the Variation is weſterly.

EXAMPLE II.

Suppoſe the Sun's true Azimuth N 83°.20′ E, but the magnetical one N 70°.30′ E. Required the Variation.

True Azimuth	N	83°.20′	E
Magnetical Azimuth	N	70. 30	E
Variation		12. 50	E

The true Azimuth being to the right of the magnetical one, the Variation is eaſterly.

EXAMPLE III.

Suppoſe the Sun's true Azimuth were S 37°. 15′ W. and the magnetical one S 44°.20′ W. Required the Variation.

True Amplitude	S	37°.15′	W
Magnetical Azimuth	S	44. 20	W
Variation		6. 05	W

The true Azimuth being to the left of the magnetical one, the Variation is weſterly.

EXAMPLE IV.

Suppoſe the Sun's true Azimuth be S 4°. 05′ W, and the magnetical one S 3°.30′ E. Required the Variation.

True

True Azimuth	S	4°.05^{l}	W
Magnetical Azimuth	S	3.30	E
Variation		7.35	E

The true Azimuth being to the right of the Magnetical, the Variation is Eaſterly.

The Variation of the Compaſs was firſt obſerved at *London*, in the Year 1580, to be one Point of the Compaſs Eaſterly, or 11°.15^{l} E, after which Time it became leſs; for in the Year 1622 it was 6°.00 E, in 1634 it was 4°.05 E, and ſo continued to decreaſe 'till the Needle coincided with the true Meridian, and then there was no Variation; after which the Variation became Weſterly, and has ever ſince increaſed to the Weſtward; for in the Year 1672 it was 2°.30^{l} W; in the Year 1683 it was 4°.30^{l} W at *London*; in 1722 it was at *Dublin* found to be 11°.15^{l} W, and in 1751 it was there found to be 19°.00^{l} W; but how far it will continue to move more Weſterly, Time and Obſervations will probably be the only Means to diſcover.

At *Paris* in 1640, the Variation was 3.00^{l} E; in 1666 there was no Variation; but in 1681 it was 2°,30^{l} W, and ſtill continues to go on Weſterly.

How to draw a true Meridian Line to a Map, having the Variation and Magnetical Meridian given.

On any Magnetical Meridian or Parallel, upon which your Map is protracted, ſet off an Angle from the North towards the Eaſt, equal to the Degrees or Quantity of Variation, if it be Weſterly, or from the North towards the Weſt if it be eaſterly, and the Line which conſtitutes ſuch an Angle with the Magnetical Meridian, will be a true Meridian Line.

For if the Variation be weſterly, the Magnetical Meridian will be the Quantity of Variation on the Weſt Side of the true Meridian, but if Eaſterly on the Eaſt Side, therefore the true Meridian muſt be a like Quantity on the Eaſt Side of the Magnetical one, when the Variation is weſterly, and on the Weſt Side when it is eaſterly.

How to lay out a true Meridian Line by the Circumferentor.

If the Variation be weſterly, turn the Box about 'till the North of the Needle points as many Degrees from the *Flower-de-Luce* towards the Eaſt of the Box, or 'till the South of the Needle points the like Number of Degrees from the South towards the Weſt, as are the Number of Degrees contained in the Variation, and the Index will be then due North and South; therefore if a Line be ſtruct out in the Direction thereof, it will be a true Meridian Line.

If the Variation were eaſterly, let the North of the Needle point as many Degrees from the *Flower-de-Luce* towards the Weſt of the Box, or let the South of the Needle point as many Degrees towards the Eaſt, as are the Number of Degrees contained in the Variation, and then the North and South of the Box, will coincide with the North and South Points of the Horizon, and conſequently a Line being laid out by the Direction of the Index will be a true Meridian Line.

This will be found to be very uſeful in ſetting an Horizontal Dial; for if you lay the Edge of the Index by the Baſe of the Stile of the Dial, and keep the angular Point of the Stile towards the South of the Box, and allow the Variation as before, the Dial will then be due North and South, and in it's proper Situation; provided the Plane upon which it is fixed be duly horizontal, and the Sun be South at Noon; but in Places where it is North at Noon, the Angular Point of the Index muſt be turned to the North.

How Maps may be traced by the Help of a true Meridian Line.

If all Maps had a true Meridian Line laid out upon them, it would be eaſy by producing it, and drawing Parallels, to make out Field Notes; and by knowing the Variation, and allowing it upon every Bearing and having the Diſtances, you would have Notes ſufficient for a Trace. But a true Meridian Line is ſeldom to be met with, therefore we are obliged to have Recourſe to the foregoing Method. It is therefore adviſeable to lay out a true Meridian Line upon every Map.

A TABLE *of the Sun's Declination.*

Days	Jan. S.	Feb. S.	March S•	April N.	May N.	June N.	Days
	° ′	° ′	° ′	° ′	° ′	° ′	
For the Years 1753, 1757, 1761, 1765, 1769, 1773.							
4	22.43	16.08	S 6.19	5.48	16.03	22.29	4
8	22.13	14.54	4.46	7.18	17.10	22.54	8
12	21.37	13.36	3.12	8.47	18.12	23.12	12
16	20.54	12.14	1.37	10.13	19.10	23.23	16
20	20.04	10.49	0.03	11.36	20.03	23.29	20
24	19.09	9.21	N 1.32	12.57	20.49	23.27	24
28	18.08	7.51	3.06	14.14	21.31	23.19	28
For the Years 1754, 1758, 1762, 1766, 1770, 1774.							
4	22.45	16.13	S 6.25	5.42	15.58	22.27	4
8	22.16	14.58	4.52	7.13	17.05	22.53	8
12	21.39	13.40	3.18	8.41	18.08	23.11	12
16	20.57	12.19	1.43	10.08	19.06	23.23	16
20	20.07	10.54	0.09	11.31	19.59	23.29	20
24	19.12	9.26	N 1.26	12.52	20.47	23.27	24
28	18.11	7.54	3.00	14.09	21.29	23.20	28
For the Years 1755, 1759, 1763, 1767, 1771, 1775.							
4	22.46	16.17	S 6.31	5.34	15.54	22.26	4
8	22.17	15.03	4.58	7.07	17.02	22.51	8
12	21.42	13.45	3.24	8.36	18.05	23.10	12
16	20.59	12.24	1.49	10.03	19.03	23.23	16
20	20.10	10.59	0.14	11.27	19.56	23.28	20
24	19.16	9.32	N 1.20	12.47	20.44	23.28	24
28	18.15	8.02	2.54	14.05	21.66	23.21	28
For the Years 1756, 1760, 1764, 1768, 1772, 1776.							
4	22.48	16.21	S 6.13	5.54	16.07	22.31	4
8	22.20	15.08	4.40	7.24	17.14	22.55	8
12	21.44	13.50	3.06	8.53	18.16	23.13	12
16	21.02	12.29	1.31	10.19	19.14	23.24	16
20	20.14	11.04	N 0.03	11.42	20.06	23.29	20
24	19.19	9.37	1.38	13.02	20.52	23.27	24
28	18.19	8.08	3.12	14.19	21.34	23.18	28

A TABLE *of the Sun's Declination.*

Days	July N.	Auguſt N.	Sept. N*	Oct. S.	Nov. S.	Dec. S.	Days
	° ′	° ′	° ′	° ′	° ′	° ′	
Each being the firſt Year after Leap-Year.							
4	22.55	17.15	N 7.09	4.26	15.29	22.20	4
8	22.30	16.09	5.39	5.58	16.41	22.48	8
12	22.00	14.58	4.08	7.30	17.48	23.09	12
16	21.43	13.44	2.35	8.59	18.51	23.23	16
20	20.41	12.26	1.02	10.27	19.48	23.29	20
24	19.53	11.05	S 0.32	11.52	20.39	23.27	24
28	19.00	9.41	2.06	13.14	21.24	23.18	28
Each being the ſecond Year after Leap-Year.							
4	22.56	17.19	N 7.14	4.20	15.25	22.17	4
8	22.32	16.13	5.44	5.53	16.37	22.46	8
12	22.02	15.02	4.13	7.24	17.44	23.08	12
16	21.26	13.48	2.41	8.54	18.47	23.22	16
20	20.44	12.31	1.07	10.21	19.45	23.28	20
24	19.57	11.10	S 0.26	11.47	20.36	23.27	24
28	19.04	9.46	2.00	13.09	21.22	23.19	28
Each being the third Year after Leap-Year.							
4	22.57	17.23	N 7.20	4.15	15.20	22.16	4
8	22.34	16.17	5.50	5.47	16.33	22.45	8
12	22.04	15.07	4.19	7.19	17.41	23.07	12
16	21.28	13.53	2.46	8.58	18.43	23.21	16
20	20.46	12.36	1.13	10.16	19.41	23.28	20
24	20.01	11.15	S 0.20	11.42	20.33	23.28	24
28	19.07	9.51	1.54	13.04	21.19	23.20	28
Each being Leap-Year.							
4	22.54	17.18	N 7.03	4.32	15.34	22.22	4
8	22.29	16.04	5.33	6.05	16.46	22.49	8
12	21.58	14.53	4.01	7.36	17.53	23.10	12
16	21.21	13.39	2.29	9.05	18.55	23.23	16
20	20.38	12.21	0.55	10.33	19.52	23.29	20
24	19.50	10.59	S 0.38	11.58	20.42	23.27	24
28	18.56	9.35	2.12	13.19	21.27	23.17	28

To reduce one MEASURE to another.

PROB. I.

To reduce Iriſh to Engliſh Acres, and the contrary.

IT is plain, that if any Parcel of Ground be meaſured by Chains of different Lengths, that the Figures conſtructed from theſe different Meaſures will be unequal, but like to each other; and therefore ſuch Figures will be in a Duplicate Proportion to their homologous Sides, by Theo. 23. Sect. 1.

Let (in Plate 2. Fig. 6) *abcde* be a Map of Ground that was meaſured by an *Iriſh* or Plantation Chain, and ABCDE be another of the ſame Ground that was meaſured by an *Engliſh* or Statute Chain; if the Line *ab* in the firſt be 11 *Iriſh* Perches, the Line AB in the other will be 14 *Engliſh* ones; for it has been already ſhewn in Page 135, that 11 *Iriſh* Perches are equal in Length to 14 *Engliſh* ones. Whence it is plain that

1. *If you would reduce Iriſh Acres to Engliſh ones, it will be*

As the Square of 11, *or* 121,
Is to the Area in Iriſh Acres,
: : *The Square of* 14, *or* 196,
To the Area in Engliſh Acres.

2. *And if you would reduce Engliſh Acres to Iriſh ones, it will be*

As the Square of 14, *or* 196,
Is to the Area in Engliſh Acres,
: : *The Square of* 11, *or* 121,
To the Area in Iriſh Acres.

EXAMPLE I.

```
        A.   R.  P.
Reduce 246.  3.  14 Plantation Meaſure to Statute
Meaſure   4
       ----
        987
         40
       ----
      39494
       ----
```

```
              P.
121 : 39494 : : 196
         196
      ------
      236964
     355446
     39494       160
     -----------              A.  R.  P.
121)7740824(63973(399. 3. 13
     ---       ---
     480       1597
     -----     -----
     1178      1573
     -----     -----
      892    40)133(3R
      ---       ---
       454       13(P
       ---       ---
        91
        ---
```

Anſwer 339A. 3R. 13P. Statute Meaſure.

EXAMPLE

EXAMPLE II.

A. R. P.
Reduce 416. 2. 14 Stat. Meaſure to Plantation Meaſure
4
1666
40
66654

P.
196 : 66654 : : : 121
121
66654
133308
66654 160
A. R. P.
196)8065134(41148(257. 0. 28.
225 914
291 1148
953 28(P.
1694
126

Anſwer 257A. 0R. 28P. Plantation Meaſure.

PROB. II.

How to reduce Plantation Meaſure to Cuningham Meaſure, and the contrary.

Since 7 Yards or 28 Quarters make an Iriſh, and 6¼ Yards or 25 Quarters a Cuningham Perch; therefore 25 Iriſh Perches, are equal in Length to 28 Cuningham ones: For 25 x 28 = the Quarters in 25 Iriſh Perches, will be = 28 x 25, the Quarters in 28 Cuningham Perches. Then as before

1. *If you would reduce Iriſh Acres to Cuningham ones, it will be*

As the Square of 25, *or* 625,
Is to the Area in Iriſh Acres,
: : *The Square of* 28, *or* 784,
To the Area in Cuningham Acres.

2. *If you would reduce Cuningham Acres to Iriſh ones, it will be*

As the Square of 28, *or* 784,
Is to the Area in Cuningham Acres,
: : *The Square of* 25, *or* 625,
To the Area in Iriſh Acres.

EXAMPLE I.

A. R. P.
In 321. 3. 20 Iriſh, how many Cuningham Acres?

A. R. P.
321. 3. 20 = 51500

P P.
625 : 51500 : : 784 : 64602

P. A. R. P.
64602 = 403. 3. 02 Anſwer.

EXAMPLE II.

A. R. P.
In 403 3. 02 of Cuningham Meaſure, how many Iriſh Acres?

A. R. P. P.
403. 3. 02 = 64602

P. P.
784 : 64602 : : 625 : 51500

P. A. R. P.
51500 = 321. 3. 20 Anſwer.

PROB. III.

How to reduce Statute Meaſure into Woodland Meaſure, and the contrary.

Since 5½ Yards or 11 half Yards make a Statute Perch, and 6 Yards or 12 half Yards, one of Woodland Meaſure; therefore 12 Perches of Statute Meaſure will be equal in Length to 11 of Woodland Meaſure. Then

1. *If you would reduce Statute Acres to thoſe of Woodland, it will be,*

As the Square of 12, *or* 144,
Is to the Area in Statute Acres,
: : *The Square of* 11, *or* 121,
To the Area in Woodland Acres.

2. *If you would reduce Woodland Acres to Statute ones, it will be,*

As the Square of 11, *or* 121,
Is to the Area in Woodland Acres,
: : The Square of 12, *or* 144,
To the Area in Statute Acres.

EXAMPLE I.

A. R. P.
In 206. 2. 12 Statute Meaſure, how much of Woodland Meaſure?

A. R. P. P.
206. 2. 12 = 33052

P. P.
144 : 33052 : : 121 : 27773

P. A. R. P.
27773 = 173. 2. 13 Anſwer.

EXAMPLE II.

A. R. P.
In 173. 2. 13 of Woodland Meaſure, how many Statute Acres?

A. R. P. P.
173. 2. 13 = 27773

P. P.
121 : 27773 : : 144 : 33052

P. A. R. P.
33052 = 206. 2. 12 Anſwer.

PROB.

PROB. IV.

How to reduce Statute Meaſure to Churchland Meaſure, and the contrary.

This is done by the ſame Proportions uſed for Statute and Plantation Meaſure, the Length of the Perch for Churchland Meaſure being the ſame with the Plantation Perch.

PROB. V.

How to reduce Statute to Foreſt Meaſure, and the contrary.

Becauſe 5½ Yards or 11 half Yards make a Statute Perch, and 8 Yards or 16 half Yards a Perch of Foreſt Meaſure; therefore 16 Perches of Statute Meaſure, will be equal in Length to 11 Perches of Foreſt Meaſure; and then,

1. *If you would reduce Statute Acres, to thoſe of Foreſt Meaſure, it will be*

As the Square of 16, *or* 256,
Is to the Area in Statute Acres,
: : *The Square of* 11, *or* 121,
To the Area in Foreſt Acres.

2. *If you would reduce Foreſt Acres to Statute ones, it will be*

As the Square of 11, *or* 121,
Is to the Area in Foreſt Acres,
: : *The Square of* 16, *or* 256,
To the Area in Statute Acres.

In 200 Statute Acres, how many Foreſt ones?

A. A. R. P.
256 : 200 : : 121 : 94. 2. 5 Anſwer.

A. R. P.
In 94. 2. 05 of Foreſt Meaſure, how many Statute Acres?

A. R. P. P.
94. 2. 05 = 15125

P. P.
121 : 15125 : : 256 : 32000

P.
32000 = 200 Acres. Anſwer.

P R O B. VI.

How to reduce Statute to Scotch Meaſure, and the contrary.

The Engliſh Perch being $5\frac{1}{2}$ Yards, or $5\frac{3}{6}$, that is 33 ſixths of a Yard; and the Scotch one being $6\frac{1}{6}$ Yards, or 37 Sixths of a Yard; therefore 37 Statute Perches are equal in Length to 33 Scotch Perches. And then,

1. *If you would reduce Statute to Scots Acres, it will be,*

As the Square of 37, *or* 1369,
Is to the Area in Engliſh Acres,
: : The Square of 33, *or* 1089,
To the Area in Scots Acres.

2. *If you would reduce Scots to Statute Acres, it will be*

As the Square of 33, *or* 1089,
Is to the Area in Scots Acres,
: : The Square of 37, *or* 1369,
To the Area in Engliſh Acres.

EXAMPLE. I.

A. R. P.
In 96. 3. 12. Statute Meaſure, how many Scots Acres?

A. R. P. P.
96 . . 3 : 12 = 15492

P. P.
1369 : 15492 : : 1089 : 12323

P. A: R. P.
12323 = 77 : 0 : 03 Anſwer.

A. R. P.
In 77 : : 0 : : 03 Scotch Meaſure, how many Statute Acres?

A. R. P. P.
77 : : 0 : : 03 = 12323

P. P.
1089 : : 12323 : : 1369 : 15492

P. A. R. P.
15492 = 96 : : 3 : : 12 Anſwer

Three very uſeful PROBLEMS.

PROB. I.

A Map with it's Area being given, and it's Scale omitted, to be either drawn or mentioned; to find the Scale.

CAST up the Map by any Scale whatſoever, and it will be,

As the Area found
Is to the Square of the Scale by which you caſt up
: : The given Area of the Map
To the Square of the Scale by which it was laid down.

The Square Root of which will give the Scale.

EXAMPLE.

A Map whoſe Area is 126 A. 3 R. 16 P. being given; and it's Scale omitted to be either drawn or mentioned; to find the Scale.

Suppoſe this Map was caſt up by a Scale of 20 Perches to an Inch, and the Content thereby produced be 31 A. 2 R. 34 P.

As the Area found 31 A. 2 R. 34 P. = 5074 P.
Is to the Square of the Scale by which it was caſt up, that is to 20 x 20 = 400
: : *The given Area of the Map* 126 A. 3 R. 16 P. = 20296 P.
To the Square of the Scale by which it was laid down.

5074 : 400 : : 20296 : 1600 the Square of the required Scale.

Root
1600(40
16
———
)00
———

Anſwer. The Map was laid down by a Scale of 40 Perches to an Inch.

PROB. II.

How to find the true Content of a Survey, though it be taken by a Chain that is too long or too ſhort.

Let the Map be conſtructed, and it's Area found as if the Chain was of a true Length. And it will be

As the Square of the true Chain
Is to the Content of the Map
: : *The Square of the Chain you ſurveyed by*
To the true Content of the Map.

EXAMPLE.

If a Survey be taken with a Chain which is 3 Inches too long; or with one whoſe Length is 42 Feet 3 Inches, and the Map thereof be found to contain 920 A. 2 R. 20 P. Required the true Content.

As the Square of 42 F. 0 In. = *the Square of* 504 *Inches* = 254016
Is to the Content of the Map 920 A. 1 R. 20 P. = 147260 P.
: : *The Square of* 42 F. 3 In. = *the Square of* 507 *Inches* = 257049
To the true Content.

P. P.
254016 : 147260 : : 257049 : 149019

A. R. P.
160)149019(931 . 1 . 19 Anſwer.

501

219

40)59(1 R.

19 P.

P R O B. III.

How to divide Land, or to take off any given Part from a Map.

Plate XII. Fig. 1.

Let ABCD, *&c.* be a Map of Ground, containing 11 Acres, it is required to cut off a Piece as DEFGID, that ſhall contain 5 Acres.

Join any two oppoſite Stations as D and G, with the Line DG, (which you may nearly judge to be the Partition Line) and find the Area of the Part DEFG, which ſuppoſe may want 3R. 20P. of the Quantity you would cut off: Meaſure the Line DG, which ſuppoſe to be 70 Perch. Divide 3R. 20P. or 140P. by 35, the ½ of DG, and the Quotient 4 will be a Perpendicular for a Triangle whoſe Baſe is 70, and the Area 140P. Let HI be drawn parallel to DG, at the Diſtance of the Perpendicular 4, and from 3 where it it cuts the Mearing, draw a Line to D, and that Line DI will be the diviſion Line; or a Line from G to H will have the ſame Effect; all which muſt be evident from what has been already ſaid.

Some neceſſary Directions concerning Surveys in general.

If you have a large Quantity of Ground to ſurvey, which conſiſts of many Fields or Holdings, and that it be required to map and give the reſpective Contents of the ſame. It is beſt to make a Survey of the Whole firſt, and to be ſatisfied that it is truly taken, as well as to find it's Content; and as you go round the Land to make a Note on the Side of your Field-Book at every Station where the Mearing of any particular Field or Holding interſects or meets the Surround; then proceed from any one of thoſe Stations, and in your Field-Book ſay, *proceed from ſuch a Station*, and when you have gone round that Field or Diviſion, inſert the Station you cloſe at, and ſo through the whole; a little Practice can only render this ſufficiently familiar, and the Method of Protraction muſt be evident from the Field-Notes. When the whole is protracted, and you are ſatisfied of the Cloſes of the particular Diviſions, caſt up each ſeverally, and if the Sum of their Contents be equal to the Content of the whole firſt found, you may ſafely conclude that all is right.

The Protraction being thus finiſhed and caſt up; transfer it on clean Paper, Vellum, or Parchment, as before; be careful to draw your Lines with a fine Pen, write on it the Names of the circumjacent Lands, and ſet No. 1, 2, 3, 4, *&c.* in every particular Field or Diviſion; let every Tenant's particular Holding be diſtinguiſhed by a different coloured Paint being run finely along the Boundaries; let all the Roads, Rivulets, Rivers, Bridges, Bogs, Loughs, Houſes, Caſtles, Churches, Beacons, (or whatever elſe may be remarkable on the Ground) be diſtinguiſhed on the Map. Write the Title of the Map in a neat Compartment either drawn, or done from a good Copper-Plate graving, with the Nobleman's or Gentleman's Arms. Prick off one of your Parallels with the Map, and on it make a Mariner's Compaſs, and draw a *Flower de Luce* to the North, and this will repreſent the magnetical North, after which ſet off the Variation, which expreſs in Figures, and through the Center of the Compaſs;

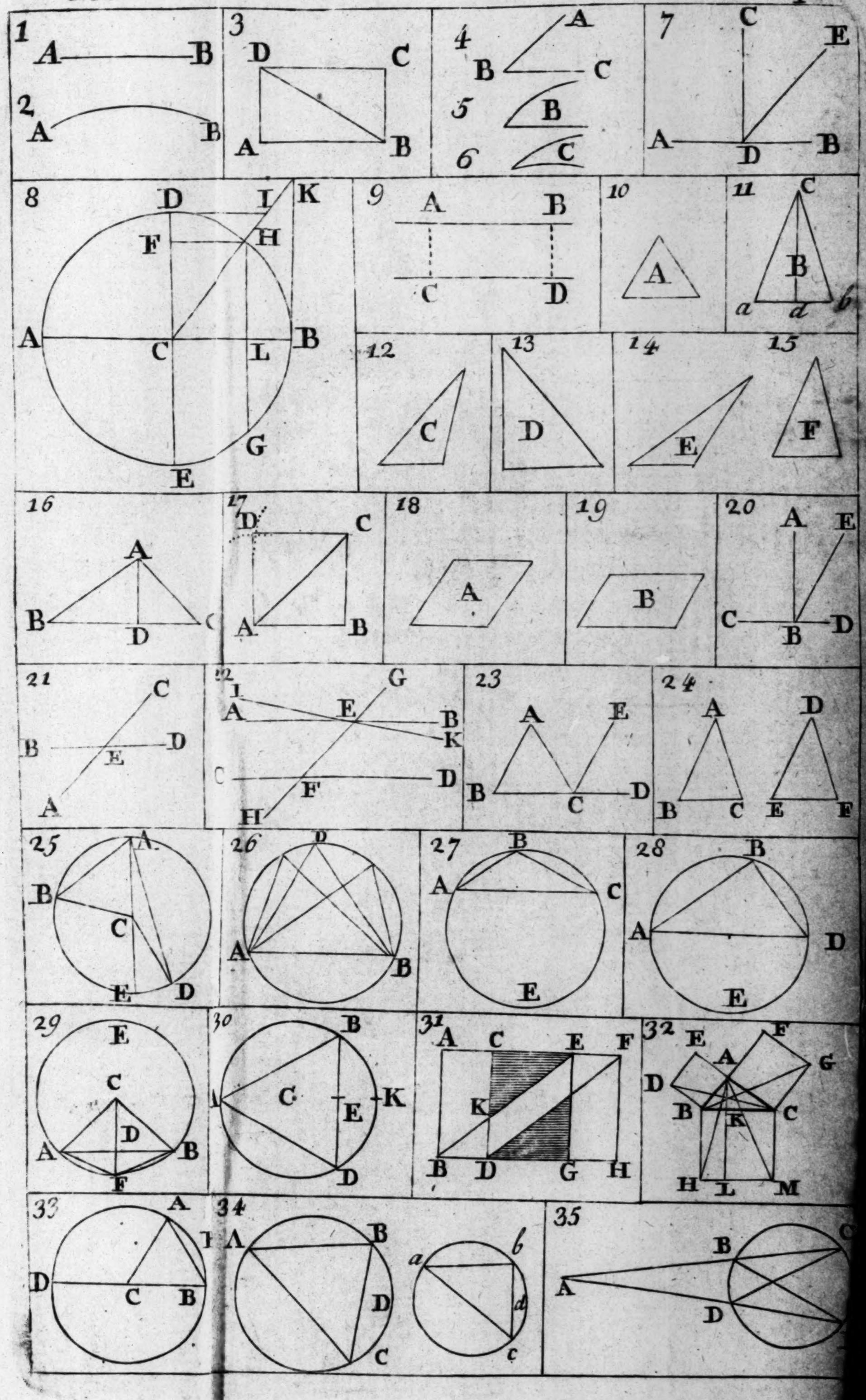
let a true meridian Line be drawn of about 3 Inches long, by which write *True Meridian.* Let a Scale be drawn, or it is ſufficient to expreſs the Number of Perches to an Inch, the Map was laid down by. Draw a Reference Table of three, or, if Occaſion be, of four or more Columns: In the firſt inſert the Number of the Field or Holding: In the next it's Name, and by whom occupied; in the third the Quantity of Acres, Roods and Perches it contains; if you have unprofitable Land, as Bog or Mountain, let the Quantity be inſerted in a fourth Column; and, if it be required, you may make another Column for Statute Meaſure, and then the Map is compleated.

It has been uſual with Writers on this Subject to treat on the ſeveral Colours fit for Maps, and of the Manner of Waſhing, Grinding, and Mixing them; but as ſuch may be had at many Druggiſts Shops, ready made up in Shells, much better than I can pretend to adviſe, I refer ſuch as want, to apply to them.

FINIS.

MVSEVM
BRITAN
NICVM

Plate 1.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3 D C A B
4 A B C
5 B
6 C
7 C E A D B
8 D I K F H A C L B G E
9 A B C D
10 A
11 C B a d b
12 C
13 D
14 E
15 F
16 A B C D
17 D C A B
18 A
19 B
20 A E C B D
21 C B D E A
22 I G A E B K C F D H
23 A E B C D
24 A D B C E F
25 A B C E D
26 D A B
27 B A C E
28 B A D E
29 E C D A B F
30 B C E K D
31 A C E F K B D G H
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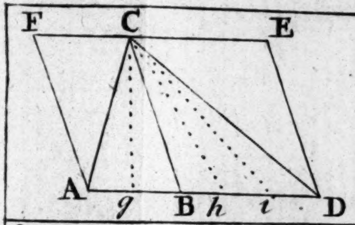
34 A B D C a b d c
35 B C A D

A

Plate 2.

II

1



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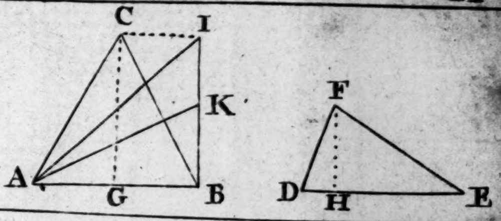

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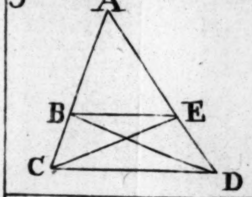

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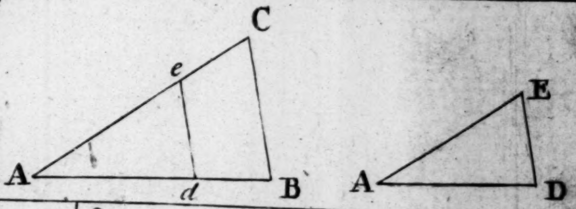

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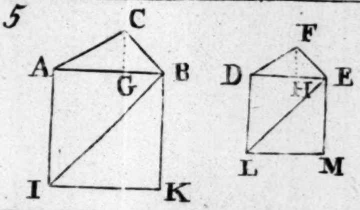

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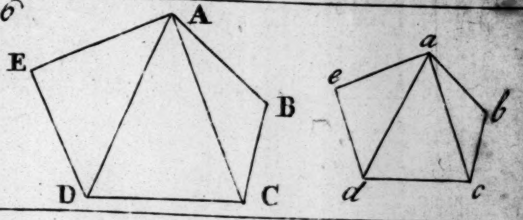

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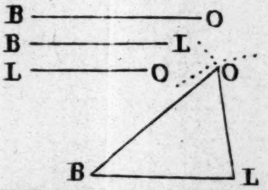

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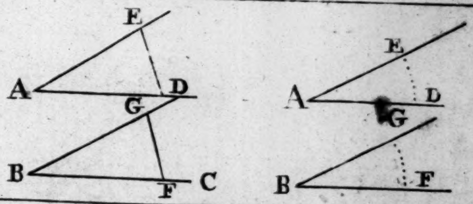

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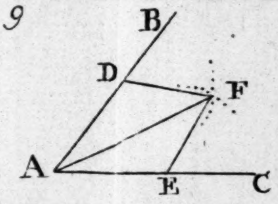

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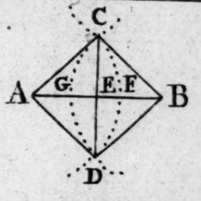

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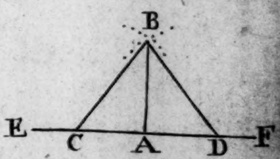

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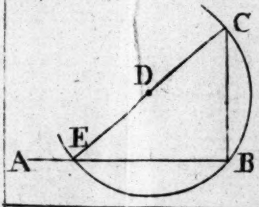

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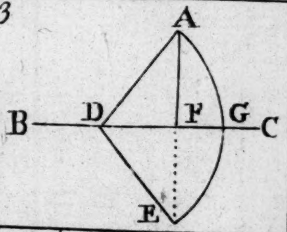

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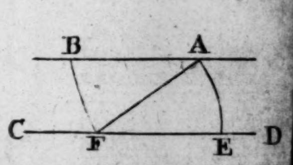

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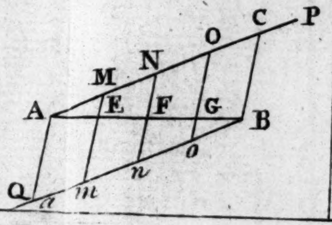

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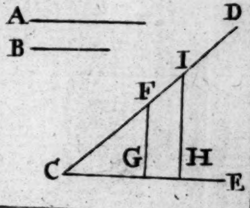

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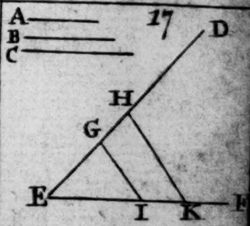

P

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E

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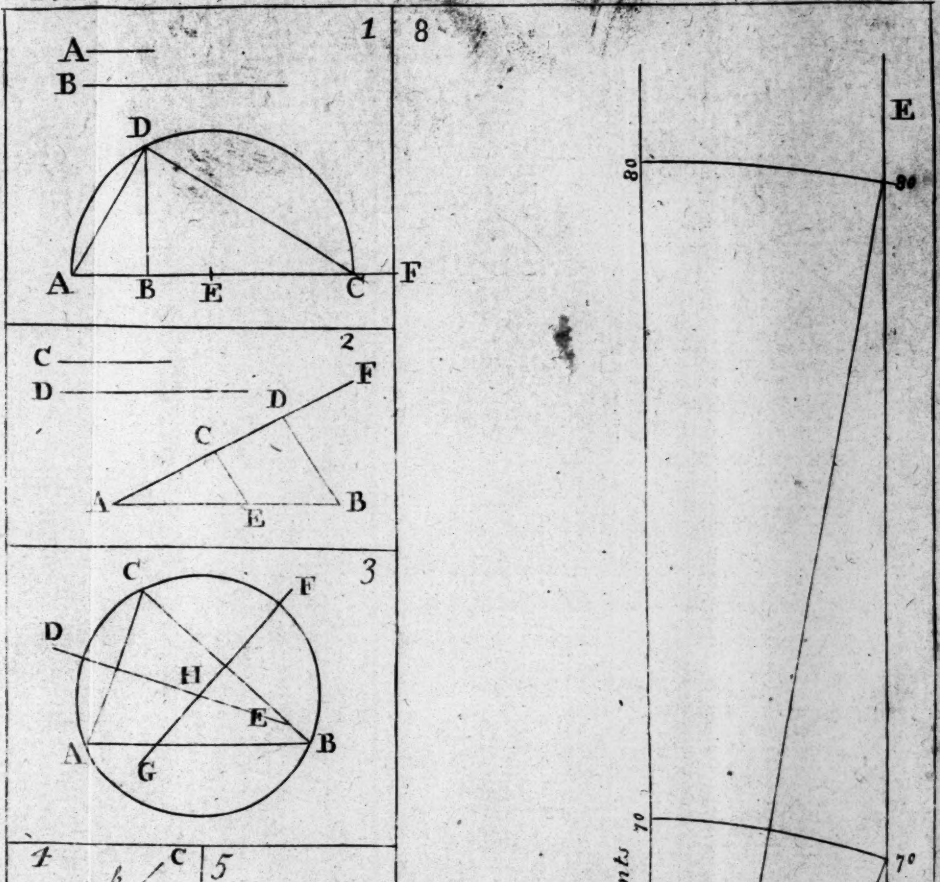
Plate III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A
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5
e
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C
d
6
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A
B
7
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C
H
A
B
F
8
E
80
80
70
70
Secants
Tangents
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A
C
B
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Sines

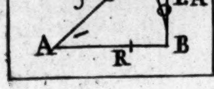
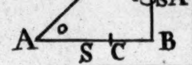
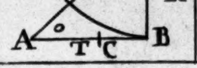
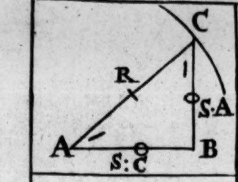
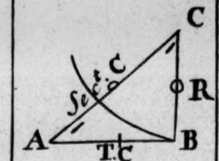
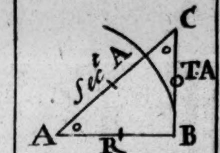
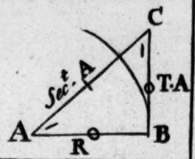
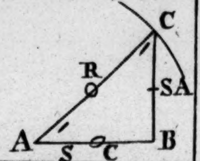
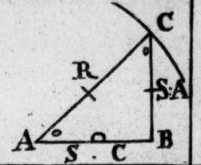
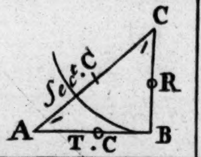
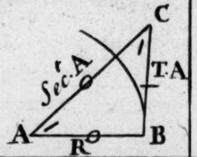
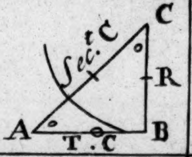
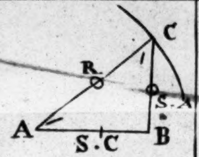
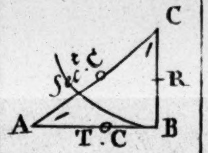
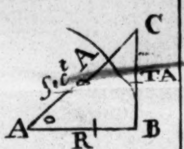
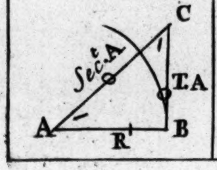
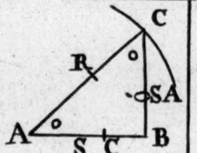
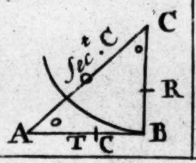
	R : AB:: *Sec*ᵗ A: A C		R: AC:: S. A: BC		R:BC::*Sec*ᵗ. C: AC

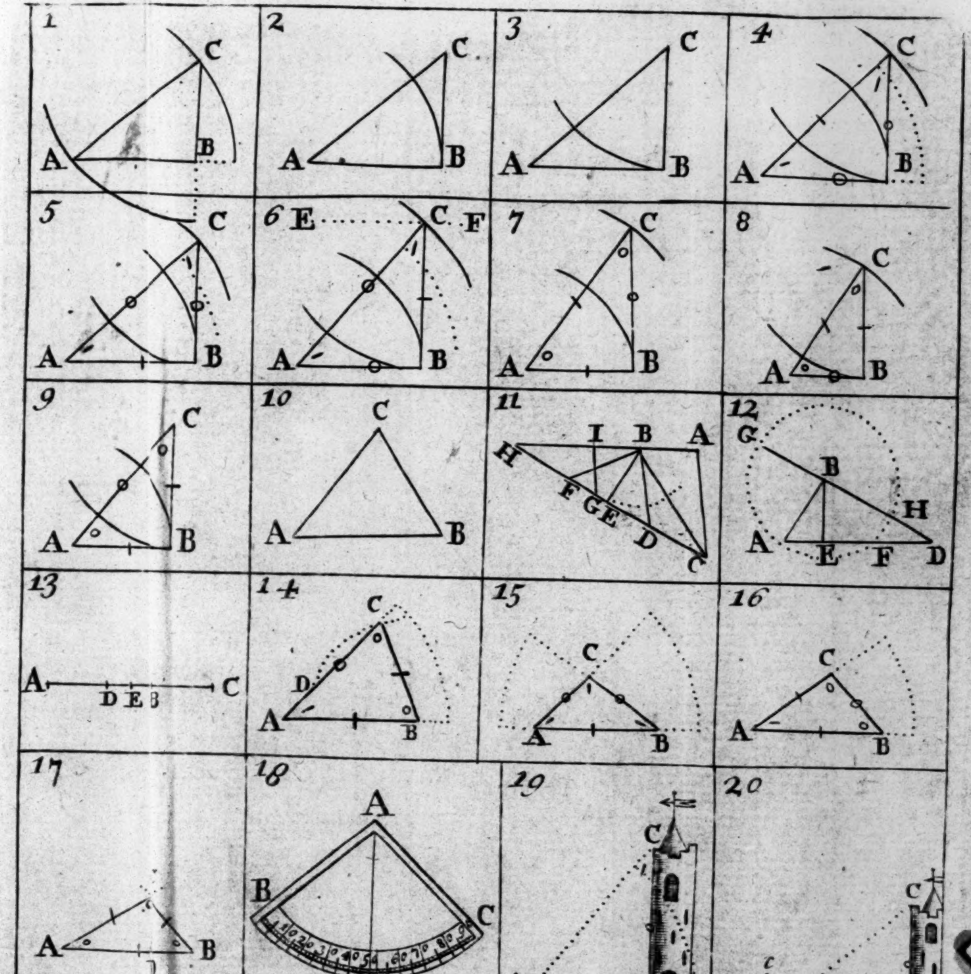
Plate IV

The Proportions for the Solution of the 6 Cases of Right Angled Plane Triangles

	*Case 1* R : AC :: S.A : BC R : AC :: S.C : AB		T.C : AB :: R : BC T.C : AB :: Sec^t C : AC		AB : R :: AC : Sec^t.A R : AB :: T.A : BC
	Sec^t A : AC :: R : AB Sec^t A : AC :: T.A : BC		*Case 3* S.A : BC :: R : AC S.A : BC :: S.C : AB		*Case 5* AC : R :: BC : S.A R : AC :: S.C : AB
	Sec^{t}C : AC :: R : BC Sec^{t}C : AC :: T.C : AB		T.A : BC :: R : AB T.A : BC :: Sec^t A : AC		BC : R :: AC : Sec^t.C R : BC :: T.C : AB
	*Case 2* S.C : AB :: R : AC S.C : AB :: S.A : BC		R : BC :: T.C : AB R : BC :: Sec^t C : AC		*Case 6* AB : R :: BC : T.A R : AB :: Sec^t. A : AC
	R : AB :: T.A : BC R : AB :: Sec^t A : AC		*Case 4* AC : R :: AB : S.C R : AC :: S.A : BC		BC : R :: AB : T.C R : BC :: Sec^t. C : AC

1
A
5
A
9
A
13
A
17
A
A
25
A

Plate V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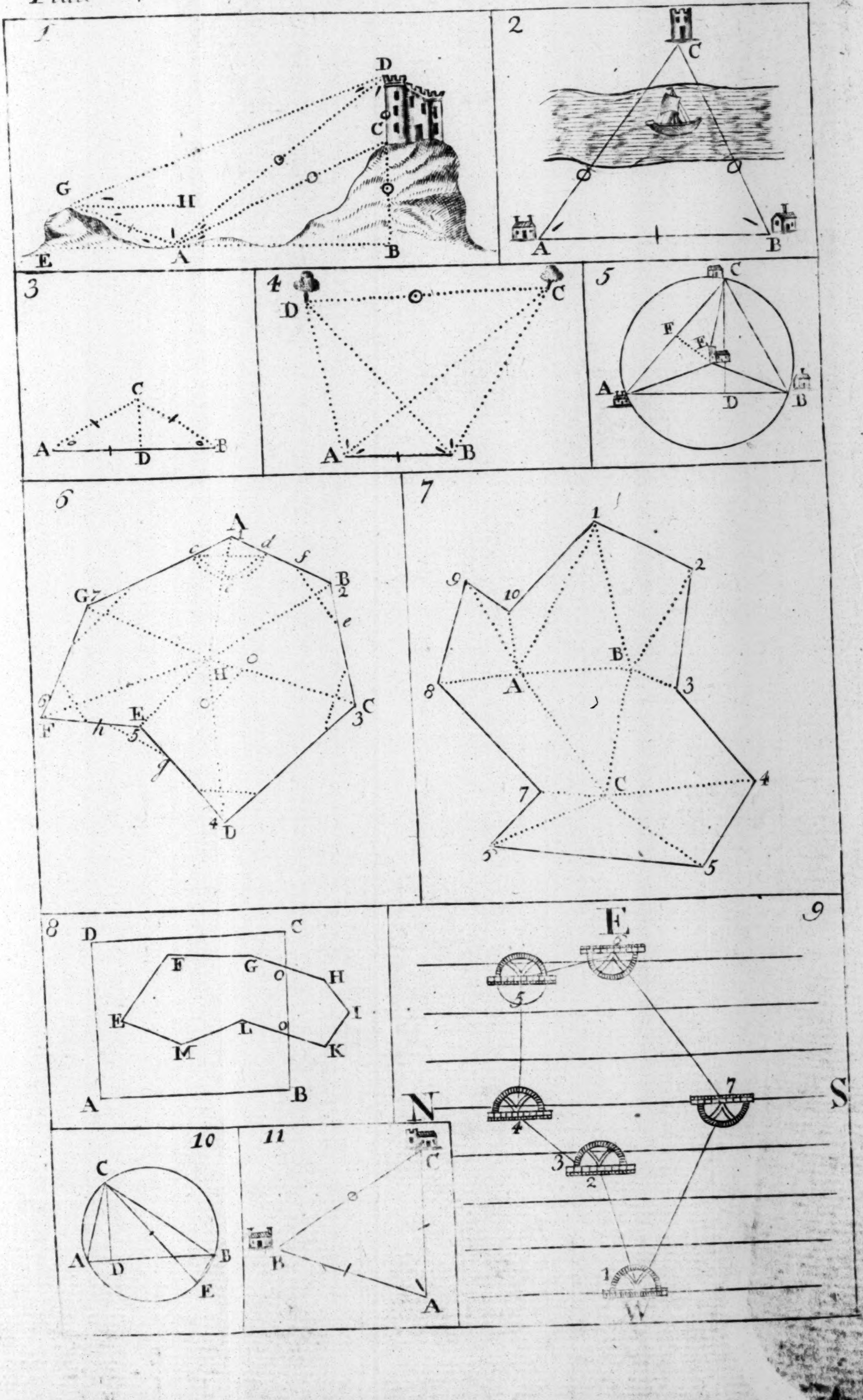
3 A B C
4 A B C
5 A B C
6 E F A B C
7 A B C
8 A B C
9 A B C
10 A B C
11 H I B A F G E D C
12 G B H A E F D
13 A D E B C
14 A B C D
15 A B C
16 A B C
17 A B D
18 A B C 10 20 30 40 50 60 70 80 90
19 A B C D
20 a b c A B C
21 A E D C B
22 A B C D
23 A B C D
24 A B C D E F G
25 A B C D E
26 A B C D E

®
th
ons

Pla

Plate VI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C
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G7
e
H
6
F
h
E
5
g
3C
4D
7
1
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9
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C
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D
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M
K
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9
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7
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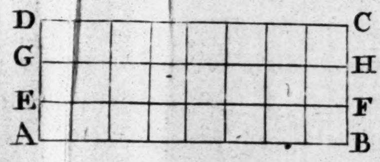
1
W
10
C
A
D
B
E
11
C
B
A

1


D
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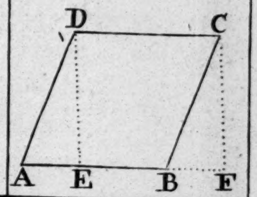
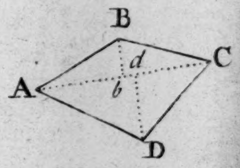
H
F
B

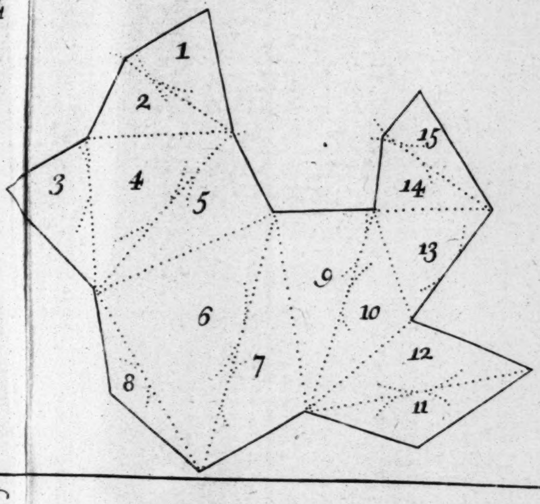
2


D
C
A
E
B
F

3


B
d
C
A
b
D

4



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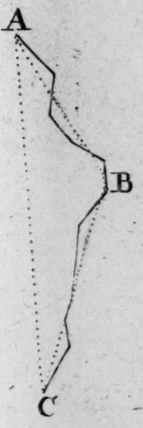
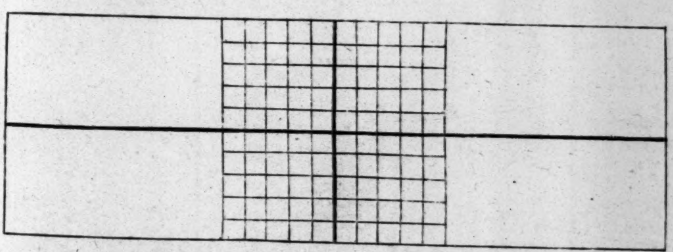

A
B
C

6



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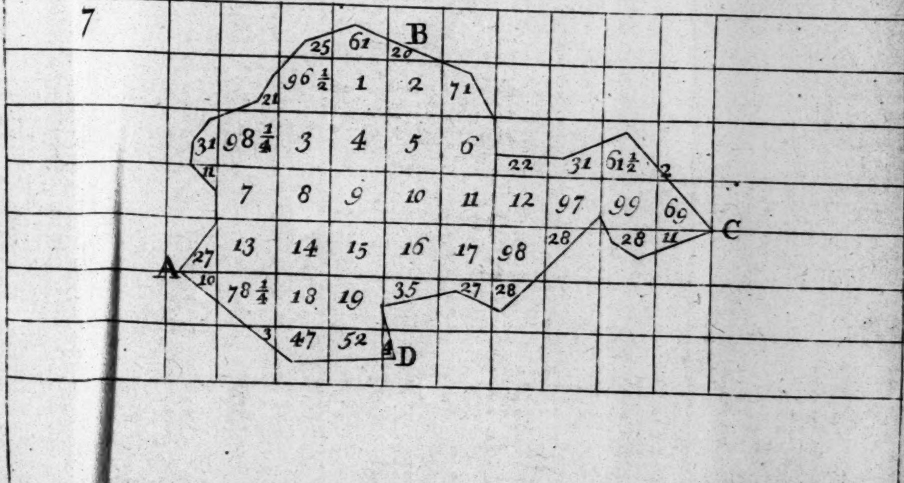
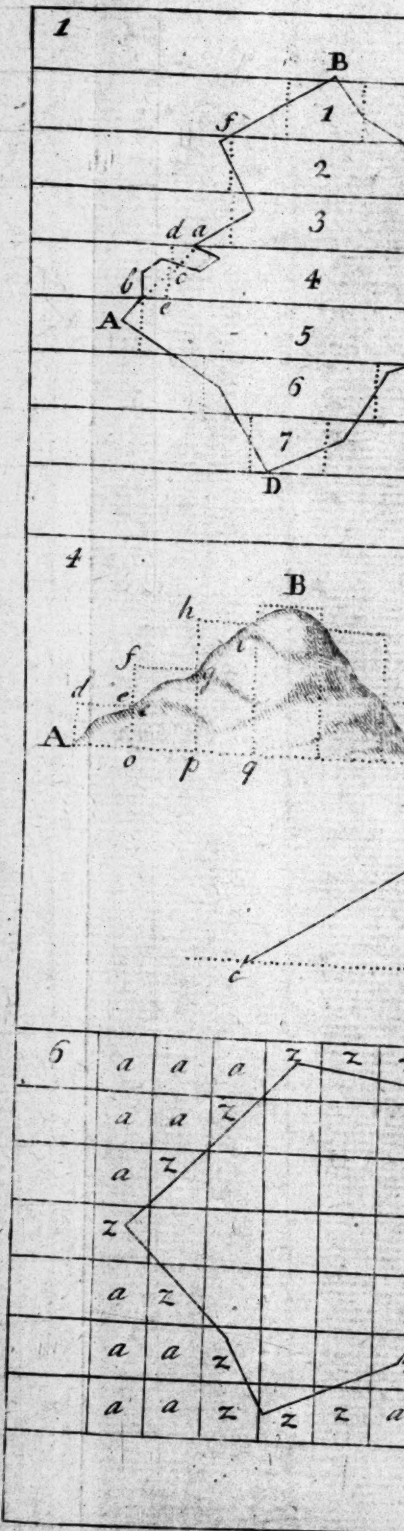

B
96 1/2
98 1/4
61 1/2
C
A
78 1/4
D

Plate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A
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D
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B
h
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a a a z z
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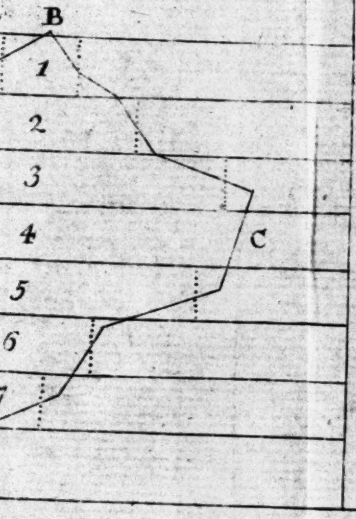
B
1
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C

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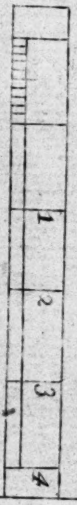
3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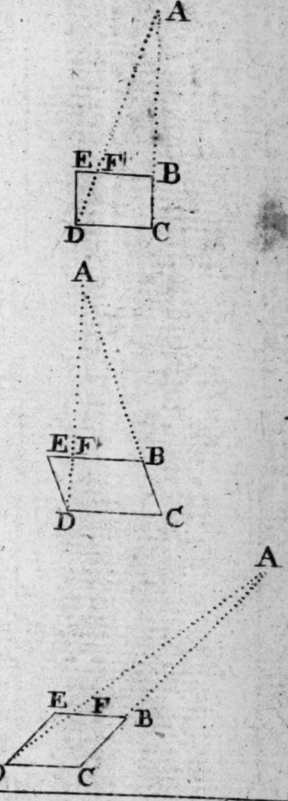
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B
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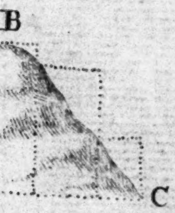
B
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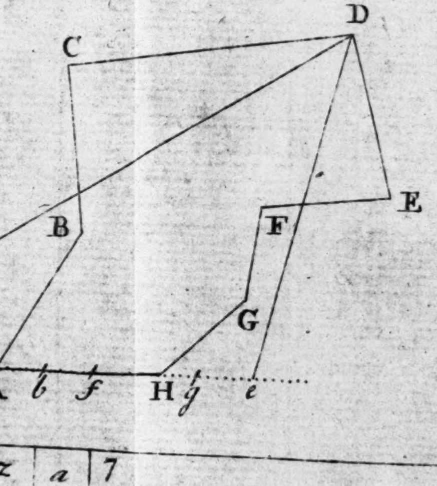
5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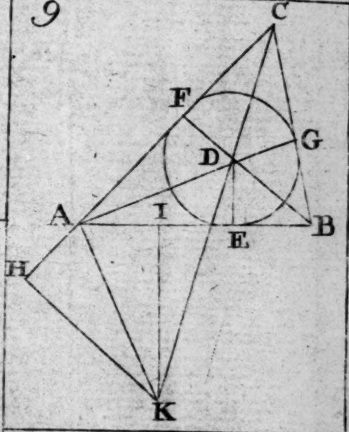
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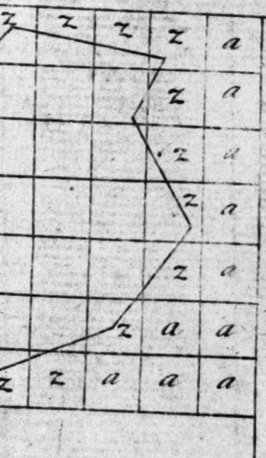
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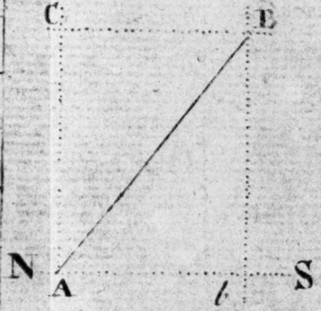
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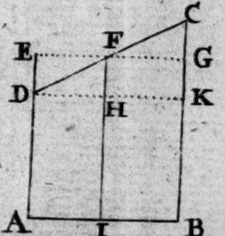
10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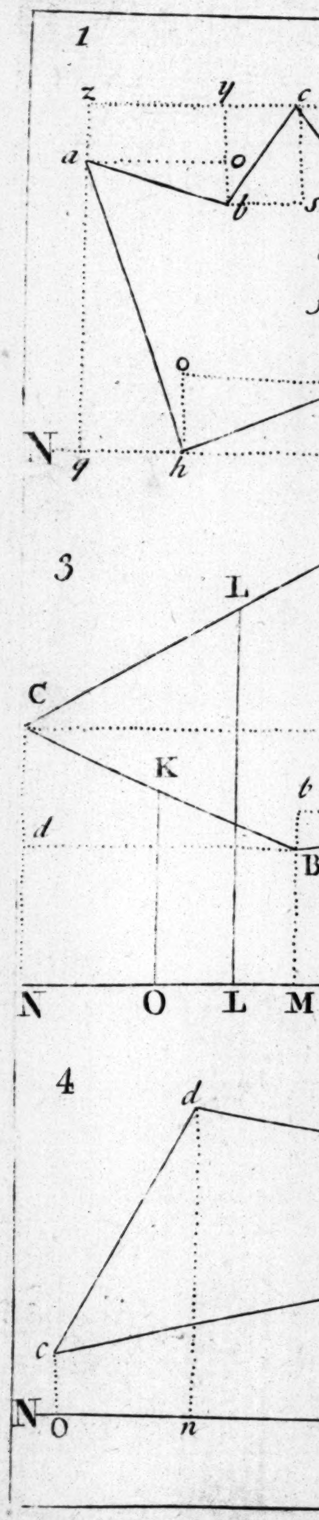

IX

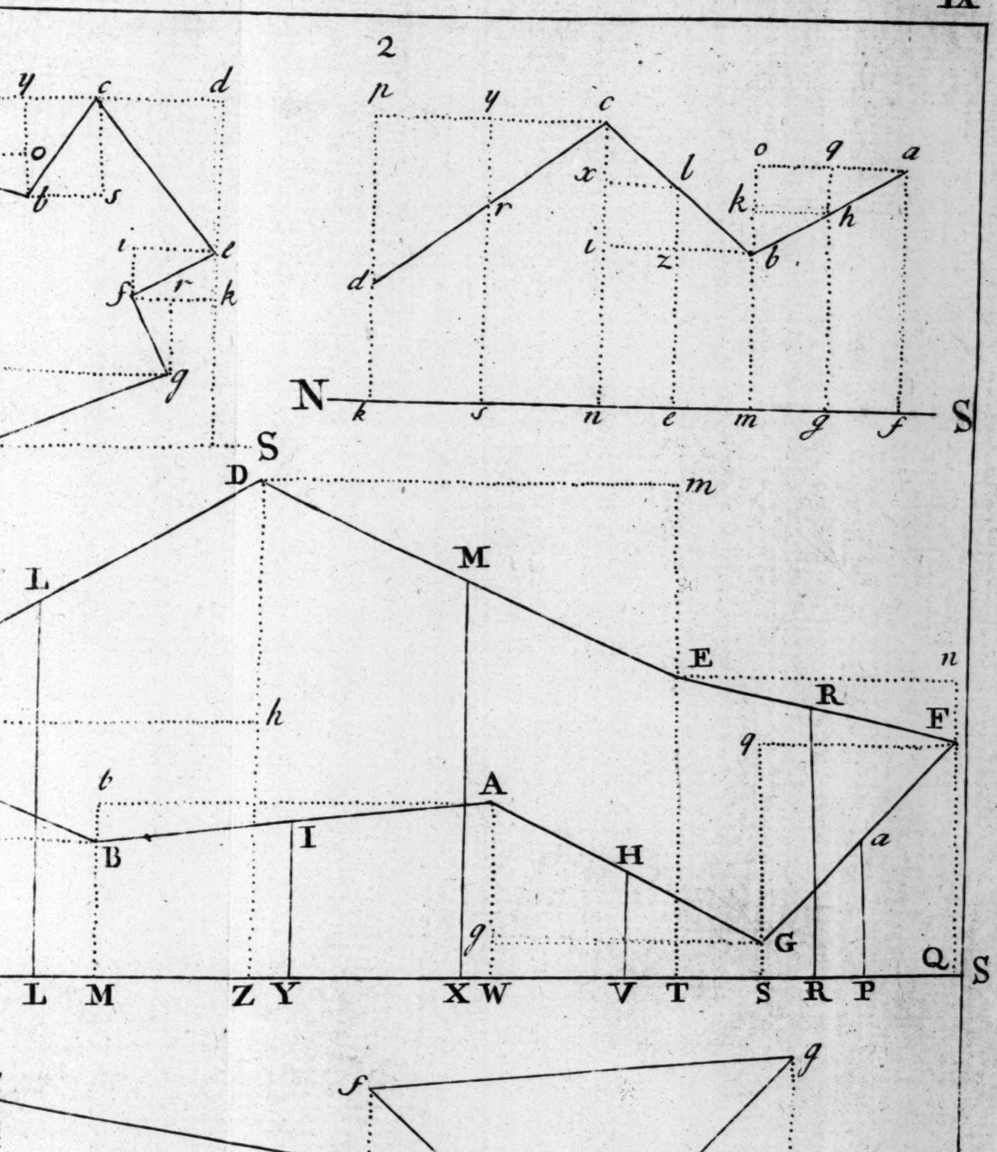
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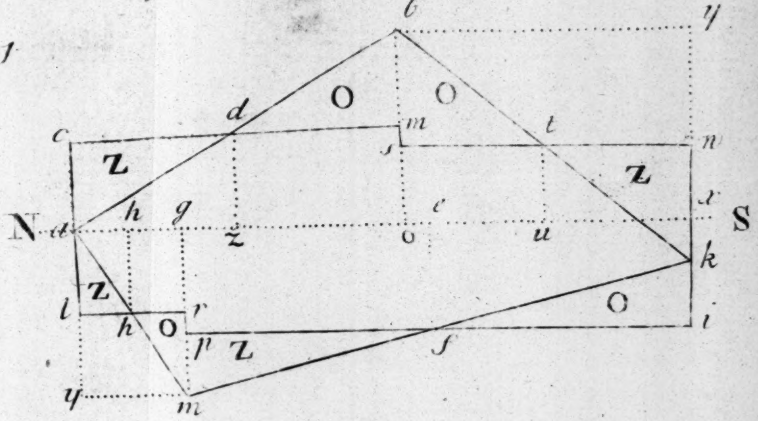
Plate X

1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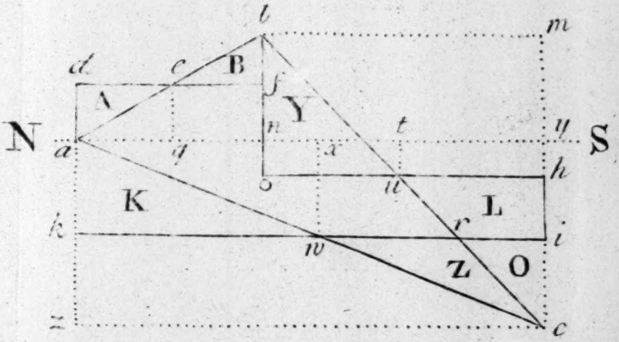
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3

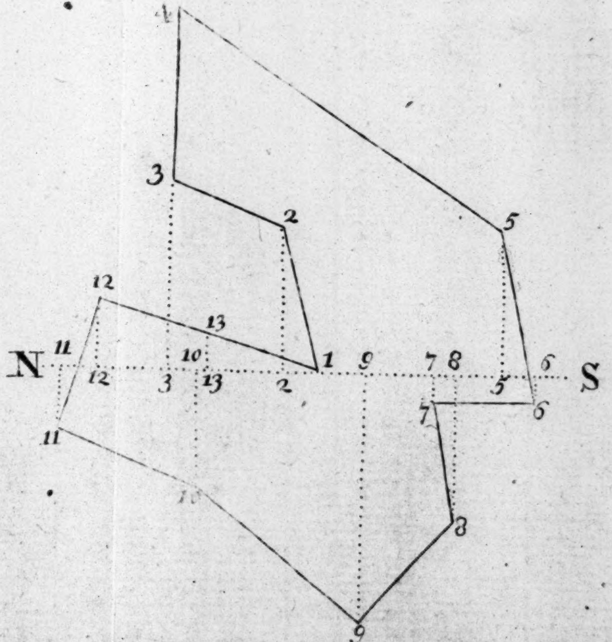
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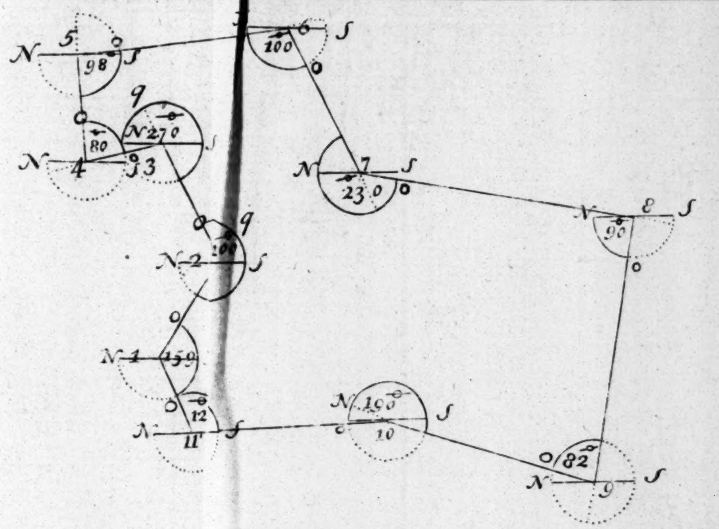
Plate XI

1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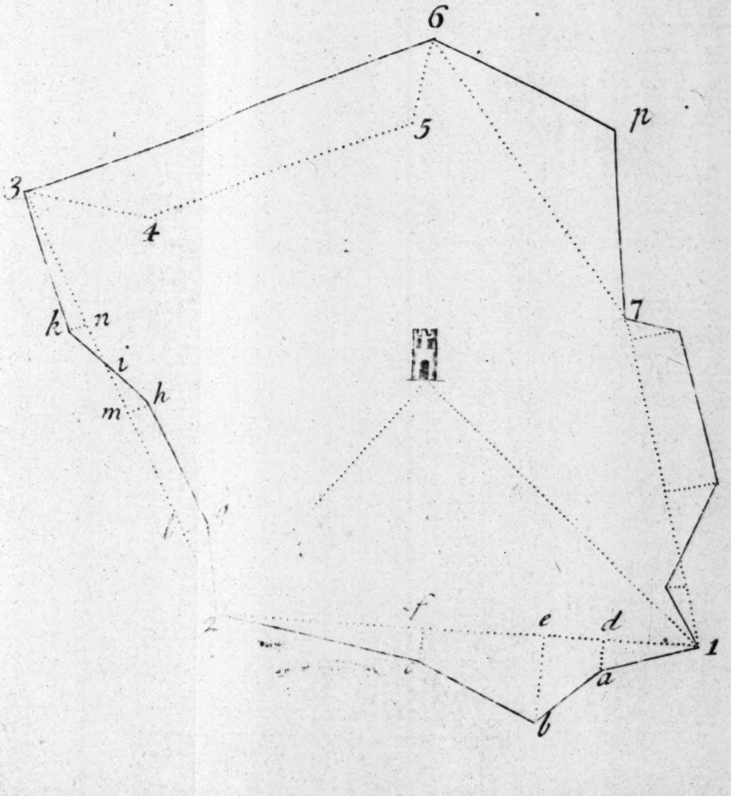
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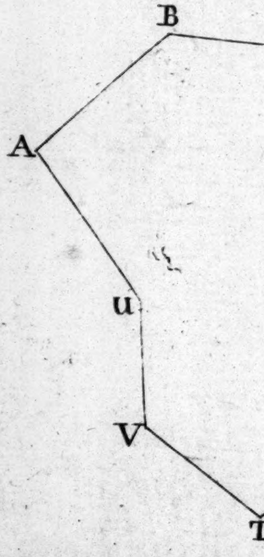
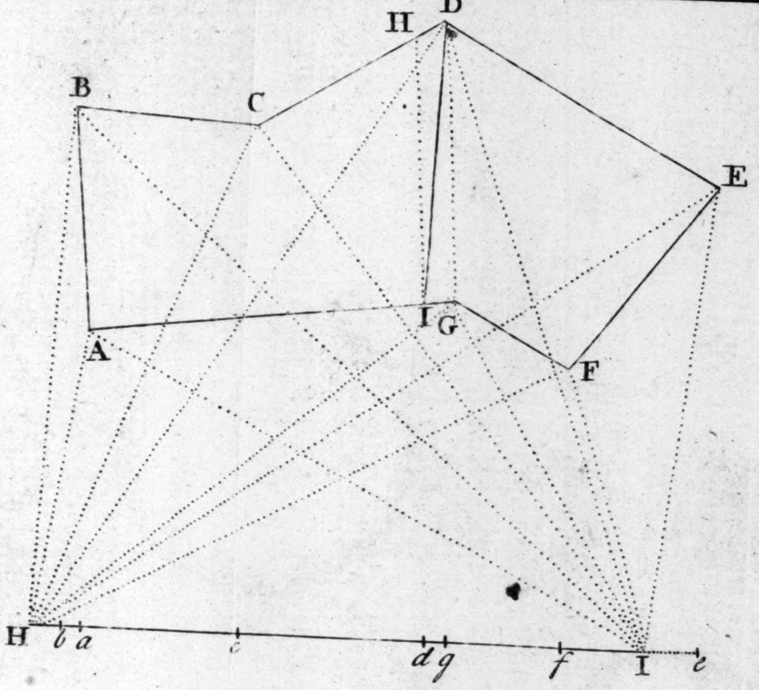
Plate



XII

1



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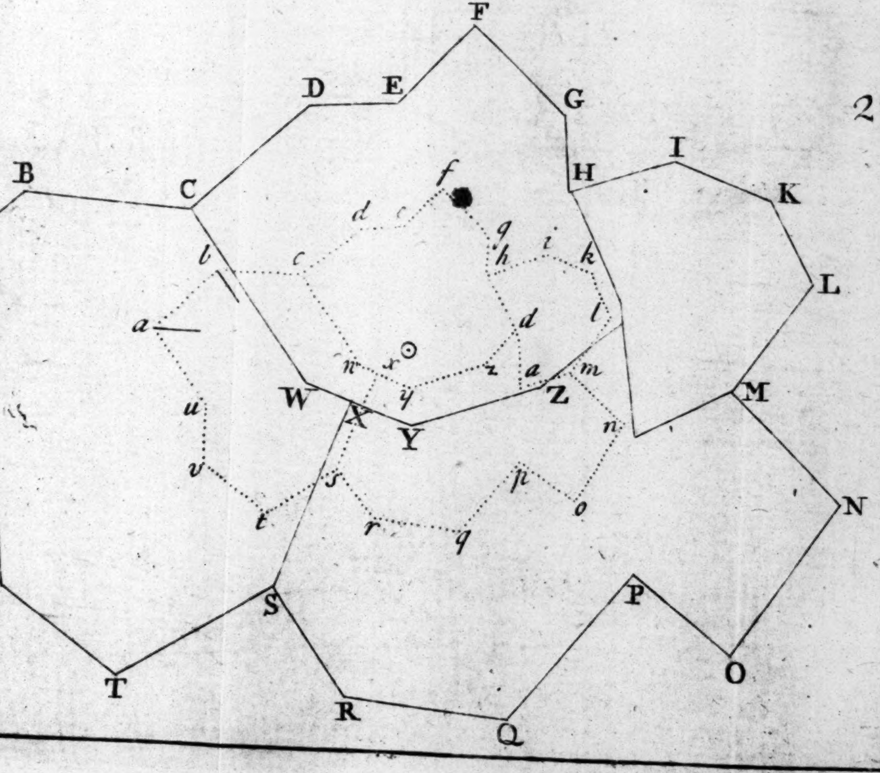

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